DOCUMENT RESUME

ED 469 440 EA 032 047

AUTHOR	Shellard, Elizabeth; Moyer, Patricia S.
TITLE	What Principals Need to Know about Teaching Math.
INSTITUTION	Educational Research Service, Arlington, VA.; National Association of Elementary School Principals, Alexandria, VA.
PUB DATE	2002-00-00
NOTE	121p.
AVAILABLE FROM	The National Association of Elementary School Principals, 1615 Duke Street, Alexandria, VA 22314 (No. PNKM: members, $19.95; nonmembers, $24.95). Tel: 703-684-3345; Tel: 800-38-NAESP (800-386-2377) (Toll Free); Web site: http://www.naesp.org; e-mail: naesp@naesp.org.
PUB TYPE	Books (010) -- Guides - Non-Classroom (055)
EDRS PRICE	EDRS Price MF01/PC05 Plus Postage.
DESCRIPTORS	Elementary Education; *Elementary School Mathematics; Instructional Materials; Mathematics Education; *Mathematics Instruction; Middle Schools; *Principals; *Secondary School Mathematics; *Teaching Methods

ABSTRACT

Designed for principals of elementary and middle schools, this guide addresses the essential elements of a high-quality mathematics program and is intended to be a practical, ongoing resource. The guide begins with an overview of the important issues in mathematics education and a discussion of the basic elements of a mathematics program. It continues with guidance on how mathematics should be taught, ways to use assessment data to inform mathematics instruction, and approaches for assisting students who are struggling in mathematics. Finally, it outlines strategies for collaborating with teachers and parents, improving teachers' skills in delivering mathematics instruction, and conducting effective observations of mathematics lessons. The guide includes checklists and other materials to use to evaluate a school's mathematics program and to collaborate with teachers and parents; lists of questions to consider when developing a plan to raise mathematics achievement; and space to write notes, reminders, and ideas. Throughout each section, the focus remains on the role of the principal in implementing and coordinating an effective program. The guide also contains a glossary and 3 appendices: Appendix A, "Children's Books with Mathematics Themes"; Appendix B, "Mathematics Mnemonics"; and Appendix C, "Handouts for Parents." (Contains 94 references.) (WFA)

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Foreword

The National Association of Elementary School Principals (NAESP) and Educational Research Service (ERS) have made a commitment to produce a series of research-based guides called *What Principals Need to Know About Teaching...* These guides are designed to better equip principals with the content-area expertise they need to serve as effective instructional leaders in their schools.

Each guide in the series focuses on an area of vital importance to effective leadership and has been developed with the busy working principal in mind. Each publication offers research-based information, advice from practitioners in the relevant field, and examples of how school leaders can apply this advice. In addition, each guide includes "how-to" tips, step-by-step suggestions, reproducible checklists and forms, and both print and online sources of further information.

This publication deals with mathematics, an area in which national and international assessments have shown U.S. students to be lagging behind their peers in other countries. Public schools need to improve the quality, rigor, and nature of the mathematics instruction we provide to our students. Principals can help in this process by remaining current on the latest research in the field and working with their teachers to ensure that they use the best research-based practices in their classrooms.

This guide will assist you in this endeavor by helping to close the divide between research and practice. We are excited about this newest guide and the prospect it holds for assisting principals in developing and implementing high-quality mathematics programs in their schools. We look forward to receiving your accounts of how you put this guide to work for you.

Vincent L. Ferrandino
Executive Director
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Elementary School Principals

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About this Guide

What Principals Need to Know About Teaching...Mathematics addresses the essential elements of high-quality mathematics programs and provides suggestions for raising mathematics achievement in the elementary and middle grades. We have designed this guide as a practical, easy-to-use resource to which you can refer on a continuing basis. In addition to a comprehensive discussion of the important issues, it includes:

- checklists and other materials to use as you evaluate your school's mathematics program and collaborate with teachers and parents;
- lists of questions to consider as you develop a plan to raise mathematics achievement; and
- space for you to write notes, reminders, and ideas.

The *What Principals Need to Know About Teaching...* series is a curricular resource to which principals can refer for concise, research-based information on the major subject areas in K-8 instruction. As the instructional leader in your building, you should be knowledgeable about effective instructional practices in each content area and able to provide teachers with sound instructional feedback and suggestions; this guide aims to promote your ability to do this. We hope that you will find this guide useful and that you will add value to it by recording your notes and observations in the provided space as you work to refine your school's mathematics programs.

We begin *What Principals Need to Know About Teaching...Mathematics* with an overview of the general issues in mathematics education and a discussion of the essential elements of high-quality mathematics programs. From here, we move on to such topics as how mathematics should be taught, ways to use assessment data to inform mathematics instruction, and approaches for assisting students who are struggling in mathematics.

Finally, we outline strategies for collaborating with teachers and parents, improving teachers' skills in delivering mathematics instruction, and conducting effective observations of mathematics lessons. Throughout each section of this guide, the focus remains on the role of the principal in coordinating the successful implementation of an effective mathematics program.

Section 1:
Issues in Math Education Today

In today's world, the mathematics needed for an intelligent citizenry goes beyond that required of our parents and grandparents. We live in a time in which mathematics is a key component of everyday life. Mathematics pervades all areas, from the workplace to the shopping mall to the kitchen. The number of mathematical decisions we make every day is astonishing—we calculate distances between locations, allot time frames in which to accomplish tasks, pay bills, calculate purchase totals, and measure and weigh quantities. All children need to understand mathematics to significantly enhance their opportunities for the future.

Although educators, policy makers, and citizens agree that every child needs to develop mathematical literacy, they often disagree on the best ways to support the development of mathematical literacy for all children. Controversies about curriculum materials, instructional strategies, and international assessment reports make it difficult for school leaders to identify those elements with the greatest potential impact on mathematical learning in their schools. In addition, there are questions about how to support the ongoing professional development of classroom teachers in using those elements to support mathematical learning for all children.

The importance of mathematical literacy is not in dispute; the question is how to achieve it for all citizens. This has led to debates at various levels within the education system.

For the past 40 years, mathematics education has experienced a conflict between "back-to-basics" proponents and the development of new practices. Innovations in the last few decades have been influenced by increased use of manipulative materials, cooperative grouping of students, increased teacher and student discourse, development of standards-based curriculum materials, problem-solving approaches, alternative forms of assessment, new developments in technology, and constructivist theories of teaching and learning.

In 1989, the National Council of Teachers of Mathematics (NCTM) took steps to clarify important goals for school mathematics instruction with the release of the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989). These landmark *Standards* documents, including *Professional Standards for Teaching Mathematics* (NCTM, 1991) and *Assessment Standards for School Mathematics* (NCTM 1995), were important catalysts for articulating common goals for effective mathematics programs.

NCTM's most recent publication, *Principles and Standards for School Mathematics* (2000), provides an update of this vision for school mathematics by outlining six key principles and five content and five process standards in four grade-level bands. The new *Standards* create a challenge to educators for teaching school mathematics in the twenty-first century.

Recent years have brought attention to the performance of our nation's children on national mathematics assessments compared with that of children in other nations. The release of local, national, or international assessment scores in mathematics often generates a great deal of discussion. Some of this discussion focuses on the validity of the testing and data collection methods that were used; other discussion centers around the causes of poor student performance. Examining why populations of students do not perform as well as other populations has become an important focus in efforts to ensure equitable opportunities for *all* children to learn mathematics.

Research on curriculum materials, instructional strategies, school reform efforts, and instructional tools (such as manipulatives, calculators, and computers) all contribute to our understanding of best practices for instruction. Many schools, however, do not provide enough opportunities for teachers to learn how to translate these findings into effective classroom practices.

The increased focus on mathematical literacy in recent years shows promise for improving mathematics instruction for all children. Discussions and debates about how to teach mathematics—although contentious at times—can also bring important ideas to the attention of teachers, parents, leaders and policymakers. Working in partnership with teachers in your school, you can ensure that the best of these ideas inform and enrich mathematics instruction for your students.

Basic Components of Mathematical Literacy

In recent years, the content of school mathematics curriculum has experienced a change in focus and content. Although computation remains a vital component, educators now recognize the importance of the ability to think critically, understand concepts and operations, and apply such knowledge to real-life situations. The following are four goals that students must achieve during elementary and middle school to be successful in mathematics:

1. *Problem solving.* Students must learn to solve problems by applying previously learned or acquired knowledge to new situations.
2. *Communicating mathematical ideas.* Students need to learn the language and notation used in the field of mathematics. They should be able to communicate mathematical ideas through the use of manipulatives, drawings, writing, and speaking.
3. *Applying mathematics to everyday situations.* Students should be able to translate daily experiences into mathematical representations such as graphs, tables, and diagrams, and interpret and explain the results.
4. *Focusing on appropriate computational skills.* Students must gain proficiency in the use of the following mathematical operations: addition, subtraction, multiplication, and division, as well as with the use of whole numbers, fractions, and decimals (Mercer and Miller undated).

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Notes, Reminders and Ideas:

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Section 2:

Essential Elements of a High-Quality Mathematics Program

Research about mathematics education has increased dramatically in recent years and has contributed greatly to our understanding of effective mathematics instruction. A variety of instructional strategies and methods have been found to improve student achievement. Many of these practices are interrelated and complement one another.

The overall vision of a mathematics program must demonstrate commitment to equity and excellence and have challenging expectations for all students. As principal, you play a key role in focusing attention on the strengths of your school's mathematics program and how it can be improved.

Effective mathematics programs contain the following essential components, which are integrated in a cohesive system:

- Teachers provide high-quality mathematics instruction.
- The mathematics curriculum is aligned with state and national standards based on research and other findings about important content and effective pedagogy.
- Teachers use appropriate mathematical tools during instruction, and the selection and use of these tools is tied to objectives that support children's learning.
- Assessment methods are integrated as a part of instruction and inform teachers' practices as well as provide feedback on individual progress.
- Instruction supports all children at different levels by challenging and motivating them to new understandings through differentiated instruction.
- The school has a coordinated plan for teaching mathematics and teachers have opportunities to grow and develop their professional skills collaboratively.

- Parents are informed about mathematics curriculum, assessment, and instructional methodologies and play an active role in their children's mathematical learning.

As the instructional leader, you are responsible for organizing and overseeing the effective implementation of these components of high-quality mathematics programs. The components are explained briefly below and discussed more fully in the following chapters.

High-Quality Mathematics Instruction

First and foremost, effective mathematics programs provide high-quality mathematics instruction focused on three important areas:

- teaching for conceptual understanding;
- developing children's procedural fluency; and
- promoting strategic competence through meaningful problem-solving investigations.

Effective instruction develops children's skills and abilities in each of these areas, rather than focusing on one area to the exclusion of another.

Conceptual understanding. During instruction, teachers must ensure that students comprehend important mathematical ideas—a skill referred to as *conceptual understanding.* To ensure that this goal is met, students should be engaged in opportunities to construct their own understanding of concepts through meaningful investigation and exploration. High-quality mathematics instruction helps children see that mathematics makes sense. For example, they should be able to look at patterns and determine how they are formed and how to extend them indefinitely. When performing a skill, children should understand *what* they are doing and *why.*

Essential Elements of High Mathematics Achievement

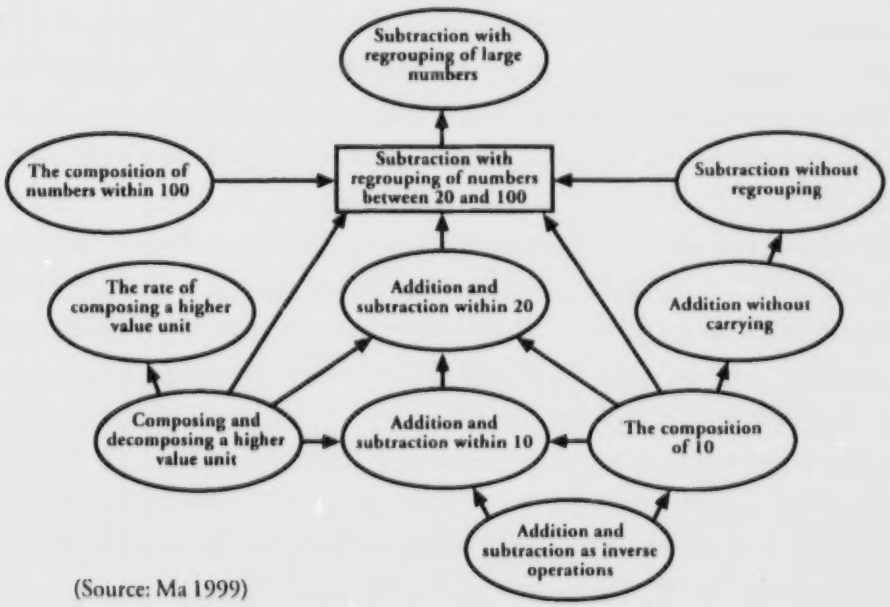
- Conceptual understanding (the ability to comprehend important mathematical ideas)
- Procedural fluency (the ability to perform basic computation quickly and accurately)
- Strategic competence (the ability to decide how to approach a problem)

To teach conceptual understanding, teachers themselves must possess a profound understanding of fundamental mathematics (Ma, 1999). Teachers' own conceptual understanding of mathematics is grounded in their knowledge of the content of mathematics; therefore, teachers who do not understand the procedures they are demonstrating for children cannot teach children to understand mathematics in a conceptual way.

Procedural fluency. High-quality mathematics instruction also develops children's ability to quickly and accurately respond to basic computational problems; a skill referred to as *procedural fluency.* Procedural fluency and conceptual understanding often go hand in hand. For example, to perform a particular procedure, such as subtraction of large numbers with regrouping, one must understand key pieces of the "knowledge package" that make up this concept, as shown in Figure 1 below (Ma, 1999). To teach this procedure effectively, teachers must understand what concepts students need to understand as the basis for the procedure.

When children understand the "why" behind the procedural steps they are performing, they can tell whether an answer is reasonable at the end of their calculations. For example, a child who divides one dollar into four parts ($1.00 ÷ 4 = □) and is satisfied with an answer of 2.5 or .025 lacks the conceptual understanding to back up his or her procedural effort. If the child understood the purpose of the task, he or she would recognize that these erroneous answers do not make sense. Such misconceptions and procedural errors are easily corrected when children have conceptual understanding as a foundation for their procedural fluency.

Figure 1: A Knowledge Package for Subtraction with Regrouping


Subtraction with regrouping of large numbers
The composition of numbers within 100
Subtraction with regrouping of numbers between 20 and 100
Subtraction without regrouping
The rate of composing a higher value unit
Addition and subtraction within 20
Addition without carrying
Composing and decomposing a higher value unit
Addition and subtraction within 10
The composition of 10
Addition and subtraction as inverse operations

(Source: Ma 1999)

Strategic competence. In addition to promoting conceptual and procedural fluency, high-quality mathematics instruction promotes children's ability to make connections and see relationships among mathematics concepts so knowledge is meaningful and useful—a skill referred to as *strategic competence.* Children who demonstrate strategic competence are able to organize their thinking to solve problems that have more than one step or that do not follow a traditional set of procedures.

Standards-Based Curriculum Materials

Although there is no national curriculum in our country, many standards-based curriculum materials do represent a common vision espoused by the National Council of Teachers of Mathematics for effective mathematics teaching and learning. The *Principles and Standards for School Mathematics* (NCTM 2000) provide a focused and coherent vision of what every student should be able to do in mathematics. These *Standards* have two main components—content standards and process standards—that can serve as a guide for selecting appropriate curriculum materials for your school.

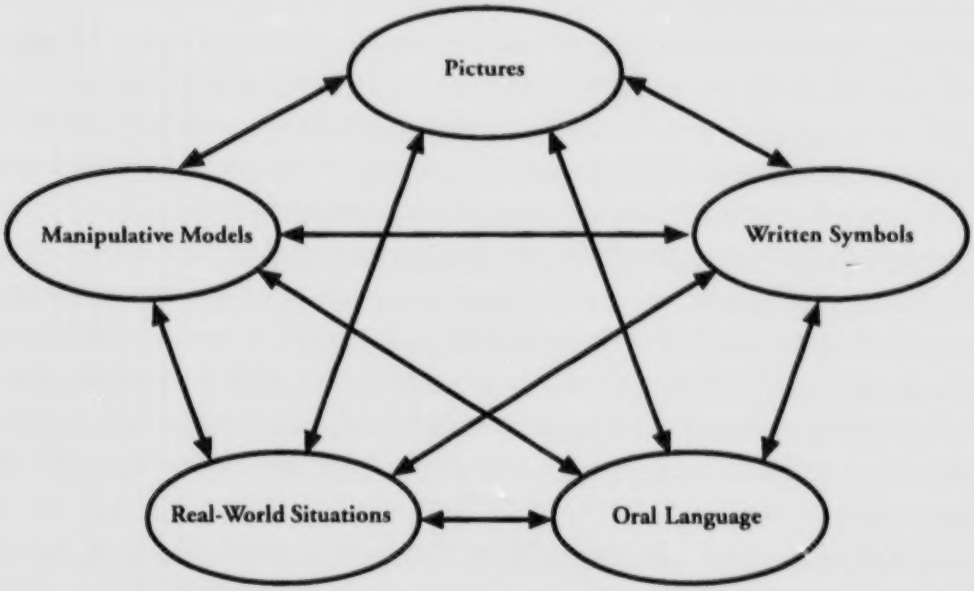
Good mathematics programs are aligned with both state and national standards (NCTM 2000) and are based on the important content and processes of mathematics. In some of the first large-scale research on new standards-based curricula, research indicates that students using the reform materials demonstrate strong procedural fluency, better understanding of concepts than students using traditional curricula do, and decreased performance gaps between majority students and poor or minority students (Schoenfeld 2002). These findings indicate that we can improve student achievement by using standards-based curricula and changing *how* we teach mathematics.

Use of Appropriate Mathematical Tools

Mathematical concepts can be represented in a variety of ways (NCTM 2000), including concrete manipulatives, pictorial models, abstract symbols, and electronic media (see Figure 2). For example, the concept of "ten" can be represented by holding up ten fingers (real-world mode), arranging ten counters or cubes (manipulative model), showing a picture of ten frogs (pictorial mode), saying the word "ten" (oral language), and writing the word "ten" or the symbol "10" (written symbols). To thoroughly understand the concept of ten, children must be able to transfer these ideas among modes of representation and to different contexts.

Concrete manipulatives are one popular mathematics teaching tool used in many elementary and middle grades classrooms. A half century ago, Piaget (1952) suggested that children do not have the mental maturity to grasp abstract mathematical concepts that are presented in words or symbols alone and need many experiences with concrete materials and drawings to learn. Because students' abstract thinking is closely anchored in their concrete perceptions of the world (Thompson 1992), actively manipulating these materials

Figure 2: Modes of Representation



(Source: Cathcart, et al. 2001)

allows learners to develop a repertoire of images that helps them mentally manipulate abstract concepts. Research has shown that the use of manipulative materials supports student achievement and improves attitudes toward mathematics (Parham 1983; Raphael and Wahlstrom 1989; Sowell 1989; Suydam 1985, 1986).

Recent research has looked at the effects of *how* teachers use manipulative materials (Moyer 2002). To increase children's learning when manipulatives are used, teachers must understand fundamental mathematics concepts and how to translate these ideas into representations and connect them with other representational forms, such as pictorial models or abstract symbols.

Assessment Tied to Objectives and Integrated with Instruction

A high-quality mathematics program integrates assessment with the teacher's instruction and uses different forms of assessment to measure student learning. Assessment is most useful when it gathers evidence from a variety of sources and is used to inform instruction and program development, in addition to providing feedback on individual students.

There are many forms of assessment teachers can use to gather this evidence. These include not only paper-and-pencil assessments (such as tests, work samples, or writing) but also observation techniques (such as recording anecdotal notes, or completing checklists on students' performance or mastery of skills), individual conferences and interviews (to gain insights into children's thinking by having them explain how they solved particular problems), performance assessments, portfolios, and journals.

Differentiated Instruction

High-quality mathematics programs provide differentiated instruction for advanced and slower learners. Teachers must work to ensure that an optimal match exists between what is being taught and each student's level and pace of learning. For advanced students, this may involve providing opportunities for extended study, independent practice, and more challenging course work.

Assisting struggling students is often an even greater challenge. Because typical classroom teachers have had limited professional development in working with students with special needs, collaboration with special education, ESL, gifted, and mathematics specialists will improve classroom instruction for these children. The specialists can suggest many different adaptations to support students who are struggling in mathematics (Ebeling, Deschenes, and Sprague 1994)—for example, cooperative learning and structured peer tutoring.

Small-group work, individual work, and whole-class instruction all play important roles in supporting students' mathematics learning. After individual and group work, whole-class discussions in which students present their work and the teacher supports them in making connections among mathematical ideas will improve student achievement. Students should have opportunities to explain their mathematical thinking both verbally and in writing in whole-group and small-group settings. These shared experiences can foster positive attitudes and confidence in mathematics.

Another strategy to support struggling students is scaffolded instruction, which supports the learner's transition from watching the teacher solve a problem or explain a concept to performing the skill independently. Using authentic contexts can also help students to make connections in mathematics.

For children with learning problems, the inability to retrieve information and strategies often impedes their ability to demonstrate what they know. Teaching learners to use metacognitive strategies often helps them remember skills that support procedural fluency. These strategies include mnemonics that help their memory, visual representations, and meaningful phrases.

Teachers can use numerous adaptations in curriculum and instruction to help learners at different levels. At the same time, however, it is important to maintain a school

vision of high expectations for excellence and the belief that all children can and will be successful in mathematics.

Ongoing Staff Development

It is often difficult for busy teachers to keep up with the latest developments in mathematics education research. Yet this research is important for understanding how children learn and for developing instructional strategies that support that learning.

As a school leader, you can support teachers' development by providing school inservice workshops, sending teachers to mathematics education conferences, and encouraging their ongoing pursuit of university credits and advanced degrees. Staff development experiences that include a balance of theory and practical knowledge are most effective in improving classroom practices. These efforts guarantee that teachers' learning about issues in mathematics is ongoing and sustained throughout their teaching careers.

You can also support teachers by scheduling time for them to meet in grade-level teams to share and develop strategies for teaching some of the essential content ideas at each grade level in mathematics. For example, a first-grade team of teachers would benefit from meeting to identify strategies for teaching addition with understanding using appropriate problems and problem solving contexts, concrete materials, and other strategies for representing these concepts. Or first- through third-grade teachers might meet to discuss their approach to mathematics instruction from a developmental perspective. Opportunities to meet and talk about mathematics teaching and learning contribute to teachers' continuous development and promote dialogue about program improvements that can lead to students' success.

Working with Parents

Collaboration with parents is critical to your school's efforts to increase student achievement and attain mathematics success for all children. Teachers can involve parents in this effort by helping parents learn *what* their children are learning in mathematics and *how* they are learning it.

There are many ways to keep the lines of communication open between the school and the home. One popular strategy is to hold a "Family Math Night," inviting parents and their children to come to the school during the evening and experience a variety of mathematics tasks together (Coates and Stenmark 1997). This is an informative way to show parents what their children are learning in school and to model questioning and discourse patterns that can help children develop mathematical thinking.

You can also keep parents informed by sending home newsletters or posting information on the school's Web site about the skill areas being addressed in the classroom.

Suggestions for working with children at home to reinforce the skills they are learning at school can accompany this information. Inviting parents for a "career day" with a focus on parents whose jobs involve mathematics shows children the real-life applications of many of the foundational skills children are learning in school. Most of all, teachers should communicate to parents the importance of mathematics for their children's future success.

In Summary

As the previous discussion indicates, an effective mathematics program requires a coordinated effort by many different stakeholders to ensure that all the components are in place. An effective mathematics program requires high-quality mathematics instruction, a curriculum aligned with state and national standards, the appropriate use of mathematical tools, varied assessments that are integrated with instruction, differentiated instruction for learners at different levels, a coordinated plan for teacher development, and the collaboration of parents. The next few sections discuss these elements in more detail.

Notes, Reminders and Ideas:

Section 3:

What Does Good Mathematics Instruction Look Like at the Classroom Level?

As a school principal, you have the important responsibility for ensuring that your school's mathematics program includes the elements of high-quality mathematics instruction. This means ensuring that teachers have the preparation necessary to teach high-quality mathematics and that support structures are in place to enable them to deliver this type of instruction. In this section, we look at the skills children need to develop to become competent mathematicians, the instructional techniques that research indicates are most effective in helping students develop these skills, and the key role of good teaching.

Balancing Instruction in Procedural Fluency with Teaching for Understanding

What is the most effective method for helping children learn mathematics? This question has been the subject of many debates. The NCTM standards advocate a form of learning that is more consistent with the developmental needs of children than traditional mathematics instruction, which focuses heavily on computation. To illustrate the difference, Wood (2001) provides the following two short interchanges between a teacher and her students. The first is typical of traditional mathematics teaching.

Tchr: [The teacher has made 45 tally marks on the board and has circled 4 groups of ten.] We have 4 groups of ten, and how many left over?

Beth: 4 tens.

Sara: 5.

Tchr: 5. Now can anybody tell me what number that could be? We have 4 tens and 5 ones. What is that number, Ann?

Ann: [remains silent]

Tchr: If we have 4 tens and 5 ones, what is that number?

Ann: 9.

Tchr: Look at how many we have there [points to the 4 groups of ten] and 5 ones. If we have 4 tens and 5 ones, we have —? [voice rises] 45.

Class: 45.

Tchr: Very good (Wood 2001, 110).

In the next interchange, the children have just discussed how they solved the problem 39+24=63 and are now discussing 39+25=___.

Tchr: Now, the next one, 39 plus 25, John and Dan.

John: We got 64.

Tchr: Explain how you got this answer.

John: Well, we know that 39 plus 20 is 59. And then 5 more would make it 64.

Tchr: All right. Let's hear from Anna and Carol. How did you solve this problem?

Anna: I did it a different way than John said. You can also go like I … [she goes to the board] these two are the same [points to the 39 in each problem]. But 24 to 25 is just one higher, so 64 (Wood 2001, 110).

The two teaching approaches are clearly different. In the first example, computation and recall of facts is the focal point, and in the second example, students are encouraged to explain how they arrived at a solution and to consider more than one way of solving a problem.

Ideally, teachers should strive for a balance between the two approaches by teaching procedural competence within a mathematics curriculum that promotes understanding and knowledge construction. Instruction should not focus on identifying one best method, but rather on providing a mixture of experiences that promote computational, conceptual, and strategic competence skills.

Understanding the Continuum of Mathematics Development

For mathematics instruction to be effective, topics must be presented in a sequence and manner appropriate for the developmental level of the students (Reys, et al. 1999). Given this requirement, it is important for both administrators and teachers to make instructional decisions on the basis of solid knowledge about how children learn mathematics.

Developmentally Appropriate Mathematics Instruction

A developmentally appropriate mathematics curriculum in grades K-4 should:

- Be conceptually oriented
- Actively involve children in doing mathematics
- Emphasize the development of children's mathematical thinking and reasoning abilities
- Include a broad range of content
- Make appropriate and ongoing use of calculators and computers

For grades 5-8, developmentally appropriate instruction consists of the following:

- Problem situations should serve as the context for mathematics
- Communication with and about mathematics and mathematical reasoning should permeate the curriculum
- A broad range of topics should be taught, with connections among them emphasized
- Technology, including calculators, computers, and videotapes, should be used when appropriate
- Learning activities should incorporate topics and ideas across mathematical areas
- Learning should engage students both intellectually and physically; students must become active learners
- Classroom activities should provide students with the opportunity to work individually and in small and large groups
- Every classroom should have ample sets of manipulative materials; resource materials on problems; and ideas for explorations, calculators, and computers.

Source: National Council of Teachers of Mathematics 1989.

Although the rate at which children develop mathematically varies from child to child, the National Council of Teachers of Mathematics (2001) has developed a general timeline for young children's mathematical skill development and instruction appropriate for each level. According to this timeline:

- From pre-kindergarten through second grade, children are developing a mathematical foundation by building beliefs about what mathematics is and what it means to know and do mathematics. They learn to understand patterns and measurement and develop a solid understanding of the numeration system.
- Instruction in grades three through five should focus on the relationship between such processes as addition and multiplication and subtraction and division. Students are introduced to multiplicative reasoning, equivalence, and several efficient and accurate methods for computing. Instruction at this level also focuses on developing children's interest in mathematics. Building on the inquisitive nature of children at this age, students are encouraged to develop and investigate solutions to everyday problems.
- Middle school students in grades six through eight are forming conclusions about their mathematical abilities, interest, and motivation that will influence how they approach mathematics in later years. Instruction at this level builds on their emerging capabilities to think hypothetically, comprehend cause and effect, and reason in concrete and abstract terms. Algebra and geometry form a large part of the curriculum during these years.

To assist teachers in planning instruction and identifying students who are struggling, lists of developmentally appropriate skills have been developed for use with children from preschool through the elementary and high school grades. The following are examples of some competencies that have been identified for children at two different grade levels:

Partial List of First-Grade Mathematics Objectives

- Count objects to 100 and write corresponding numerals. Rote count to 100 by 1's, 2's, 5's, and 10's.
- Recall basic addition facts to 10. Solve story problems involving one-step solutions.
- Tell time to the half-hour using analog or digital clocks.
- Collect and record data to create picture and bar graphs, charts, and tables. Interpret the results. Use the words *fewer, more, greater,* and *less.*
- Draw and describe plane figures according to number of sides and corners (Loudoun County Public Schools 2001, online).

Partial List of Fourth-Grade Mathematics Objectives

- Identify and compare numeration values to one million, and examine the relationship of rounding numbers to the appropriate value.
- Evaluate the relationship of addition and multiplication.
- Identify, read, compare, and estimate fractions with like and unlike denominators. Find equivalent fractions and relate fractions to decimals.
- Determine perimeter of figures using appropriate tools and units. Identify and illustrate points; rays; angles; and parallel, cross, and intersecting lines.
- Collect, analyze, and record data by creating line and bar graphs. Predict, perform, and record results of a probability experiment. Determine the probability of a given event (Loudoun County Public Schools 2001, online).

An important key to providing developmentally appropriate mathematics instruction, at any age or grade level, is achieving balance between teaching for conceptual understanding and teaching for procedural fluency. Children must first develop an understanding of the concepts they are studying before they apply these ideas to procedural strategies. Accordingly, as students are engaged in a lesson, the teacher needs to focus on those aspects that help children understand what they are doing before they are asked to perform procedural skills.

When children learn procedures without meaning, they are really only memorizing discrete pieces of information that are much more difficult for them to remember. To determine readiness, a teacher can pose questions to assess children's level of understanding at different points during the lesson, before the application of skills. Developmentally appropriate mathematics instruction ensures that children will see mathematics as a body of interrelated, connected ideas that build on one another.

Creating Effective Mathematical Environments

As principal, you should encourage teachers to improve the effectiveness of their mathematics instruction by creating effective mathematics classroom environments. Both teacher behaviors and the characteristics of classroom instruction are important.

Teacher Behaviors

In effective mathematics classrooms, you should expect to see teachers projecting a positive attitude about mathematics, using questioning techniques to facilitate learning, and encouraging children to develop divergent solutions and inventive ideas that are presented to the whole group. It is important to realize the degree to which teacher behaviors can influence children's motivation to learn mathematics.

Acceptance of Children's Divergent Ideas

Throughout a lesson, the teacher should accept different responses from children and challenge them to think more deeply about the problems they are solving. An atmosphere that is accepting and encouraging is important so children do not develop mathematics anxiety. It is important that teachers' comments and questions help children develop confidence in their own abilities to do mathematics. The environment should be one in which children learn the acceptance of divergent answers, ideas, and solution routes. This helps children view mathematics as something that can be learned by everyone.

Teacher Questioning

Teachers can influence learning by posing challenging and interesting questions. Comments and questions for children should demonstrate a sound understanding of the mathematics behind the lesson. Teachers should model appropriate mathematical language and a disposition for solving challenging mathematics problems. Rather than tell children the answers, teachers should pose questions that stimulate their curiosity and encourage them to investigate further. Teachers may give children suggestions, hints, and ask additional questions that lead them to search for different solutions. The questions should encourage children to rely on one another and themselves for ideas about mathematics.

Teacher Attitudes

Children's motivation and success in mathematics is greatly influenced by their teacher's attitude. It is important that the teacher's attitude communicates what is important to learn and how it is learned. Children observe their teacher's behaviors and verbalizations and pick up on subtle forms of communication that relay the teacher's attitudes about mathematics.

To create an environment conducive to mathematical learning, teachers must communicate to children that mathematics is an important activity and that their efforts in that activity are valued. A teacher's attitude about mathematics should demonstrate enthusiasm and a belief that all children are capable of learning mathematics. Lessons should be designed that show that the teacher's attitude toward mathematics is one of curiosity and interest and this should be evident in the types of lessons that are planned for children.

Instructional Characteristics

In addition to teacher behaviors, certain instructional characteristics are also associated with effective mathematics instruction. As you observe mathematics lessons, you should see:

- Children actively engaged in doing mathematics
- Children solving challenging problems

- Teachers using interdisciplinary connections to teach mathematics
- Children sharing their mathematical ideas while working in pairs and groups
- Teachers providing children with a variety of opportunities to communicate mathematically
- Children using manipulatives, calculators, and computers

Children Actively Engaged in Doing Mathematics

Children should not be sitting back and watching others "do mathematics;" rather, everyone should be engaged in the investigations. This engagement might take the form of working on an interesting problem, using manipulatives to find several solutions to a problem, or using a computer graphing program to represent mathematical data. What is important is that all children are involved in finding the solutions.

Children Solving Challenging Problems

Mathematics is a stimulating and interesting field that generates new knowledge every day, and children should be exposed to this excitement and challenge. Problems that involve a variety of steps or that model mathematics problem solving in the real world are more authentic and meaningful for children. Many different skills can be embedded in an interesting problem.

Interdisciplinary Connections

Using a children's book as a springboard into a mathematical investigation is a great way to introduce a problem in context and encourage meaningful mathematics investigations. The integration of children's literature into mathematical instruction can develop students' language skills as well as their mathematical literacy. There are many children's books that develop mathematical themes at a variety of grade levels (see Appendix A); a sample lesson idea can be found on the next page.

A mathematics investigation placed in the context of a story provides an authentic problem-solving situation, sometimes with "messy" results that model real-world problems. This engages children in connecting the language of mathematical ideas with numerical representations. These are important skills that support children's abilities to solve word problems. Teachers may also connect mathematics with other content areas in the school curriculum, such as science, social studies, or language arts, to show that mathematics is relevant to solve problems in other disciplines.

Opportunities for Group Work

Research shows that students who work in groups on problems, assignments, and other mathematical investigations show increases in achievement. Group activity is most effective when learners work toward group goals and are held individually accountable. Opportunities for children to work in pairs and small groups give students the chance to

Literature Link

The King's Chessboard by David Birch. Puffin (1988).

The King's Chessboard demonstrates the number pattern of exponential growth in a meaningful context and provides children with opportunities to practice problem-solving strategies. In the story, the king wishes to reward a wise old man who desires no reward from the king. To make a point, the wise old man requests grains of rice according to the number of squares (64) on the king's chessboard. For the first square, 1 grain of rice; the second square, 2 grains of rice; the third square, 4 grains of rice; the fourth square, 8 grains of rice; the fifth square, 16 grains of rice, and so on. With each successive square on the chessboard, the number of grains of rice doubles. Translating the problem in the story into a mathematical sentence is an interesting challenge for children.

- Before children solve the problem posed in the story, have them discuss their predictions on how much rice the king will need to give to the wise man.
- Use representation to record the very large numbers in the story. As children are working, ask them questions such as, "Is it possible to use tally marks to keep track of the rice the king gave in the book?" and "Can you think of a way to determine the total number of grains of rice?"
- Write several mathematical sentences to analyze and visualize the problem presented in the story. Present and discuss these mathematical expressions. Encourage the use of calculators.
- While children are working, use questions to help them focus on successful and unsuccessful solution routes. Promote their use of charts, drawings, number tallying, mathematical sentences, and other problem-solving methods that will support their attempts.
- When children are finished, encourage them to share their solution routes and attempts. What methods worked, and what did children do when a method did not work? How did they know they were headed in the wrong direction?
- Create a large chessboard bulletin board display and have children write the number of grains of rice on each square of the chessboard.

(Source: Cathcart, et al. 2001)

share their ideas and solution routes with peers. As children listen to and talk with one another, they begin to see mathematical relationships that build on their previous notions and conjectures. Giving an explanation to a peer is also positively related to increased student achievement. In addition to increasing children's mathematics achievement, group work gives children the chance to elaborate on their explanations more formally, engage in collaborative dialogue while working toward the same goal, and resolve conflicting points of view.

Opportunities for Children to Communicate

During a lesson, there should be many opportunities for children to communicate their ideas. Children may draw their ideas in pictures or write in mathematics journals. After they work on a challenging problem, the teacher should ask different students or student groups to share their solution routes by presenting these on chart paper or on a transparency. Whole-class discussions are very effective for sharing and explaining a variety of solutions. This gives children the opportunity to challenge and evaluate the validity of other children's ideas in an environment of respect and understanding. An effective mathematics lesson provides many opportunities to talk about mathematics.

Use of Manipulatives

The long-term use of mathematics manipulatives is positively related to children's achievement and attitudes about mathematics. However, it is not enough to simply provide students with manipulatives; students must be taught how to use these materials. When used correctly, manipulatives are "conducive to the concrete kinds of learning that lay a sufficient foundation for abstract thought" (Ross and Kurtz 1993, 256).

Several steps can be taken to ensure that students benefit from a lesson involving manipulatives. First, the teacher must make certain that the manipulatives support the lesson's objectives. Second, before allowing students to handle the materials, the teacher must demonstrate use of the manipulatives and procedures for handling them. And third, the lesson design must support the active participation of all students. Ross and Kurtz (1993) put it this way:

> The planning of any lesson and the choice of appropriate manipulative materials must begin with a learning objective clearly in mind. The teacher then chooses or devises a manipulative-rich activity that she or he concludes will best help students attain the desired objective. Simply making base-ten blocks available to students while they work on pencil-paper tasks is less effective than designing a lesson that employs the attributes of manipulatives to address the lesson's objectives (256).

Teachers should provide children with a wide variety of manipulative materials for

NCSM Components of Essential Mathematics

1. **Problem Solving:** Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with non-text problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. Students should see alternate solutions to problems; they should experience problems with more than a single solution.

2. **Communicating Mathematical Ideas:** Students should learn the language and notation of mathematics. For example, they should understand place value and scientific notation. They should learn to receive mathematical ideas through listening, reading, and visualizing. They should be able to present mathematical ideas by speaking, writing, drawing pictures and graphs, and demonstrating with concrete models. They should be able to discuss mathematics and ask questions about mathematics.

3. **Mathematical Reasoning:** Students should learn to make independent investigations of mathematical ideas. They should be able to identify and extend patterns and use experiences and observations to make conjectures (tentative conclusions). They should learn to use a counter example to disprove a conjecture, and they should learn to use models, known facts, and logical arguments to validate a conjecture. They should be able to distinguish between valid and invalid arguments.

4. **Applying Mathematics to Everyday Situations:** Students should be encouraged to take everyday situations, translate them into mathematical representations (graphs, tables, diagrams, or mathematical expressions), process the mathematics, and interpret the results in light of the initial situation. They should be able to solve ratio, proportion, percent, direct variation, and inverse variation problems. Not only should students see how mathematics is applied in the real world, but they should observe how mathematics grows from the world around them.

5. **Alertness to the Reasonableness of Results:** In solving problems, students should question the reasonableness of a solution or conjecture in relation to the original problem. Students must develop number sense to determine if results of calculations are reasonable in relation to the original numbers and the operations used. With the increase in the use of calculating devices in society, this capability is more important than ever.

6. **Estimation:** Students should be able to carry out rapid approximate calculations through the use of mental arithmetic and a variety of computational estimation techniques. When computation is needed in a problem or consumer setting, an estimate can be used to check reasonableness, examine a conjecture, or make a decision. Students should acquire simple techniques for estimating measurements such as length, area, volume, and mass (weight). They should be able to decide when a particular result is precise enough for the purpose at hand.

NCSM Components of Essential Mathematics

7. **Appropriate Computational Skills:** Students should gain facility in using addition, subtraction, multiplication, and division with whole numbers and decimals. Today, long, complicated computations should be done with a calculator or computer. Knowledge of single-digit number facts is essential, and using mental arithmetic is a valuable skill. In learning to apply computation, students should have practice in choosing the appropriate computational method: mental arithmetic, paper-pencil algorithm, or calculating device. Moreover, there are everyday situations that demand recognition of, and simple computation with, common fractions. In addition, the ability to recognize, use, and estimate with percents must also be developed and maintained.

8. **Algebraic Thinking:** Students should learn to use variables (letters) to represent mathematical quantities and expressions; they should be able to represent mathematical functions and relationships using tables, graphs, and equations. They should understand and correctly use positive and negative numbers, order of operations, formulas, equations and inequalities. They should recognize the ways in which one quantity changes in relation to another.

9. **Measurement:** Students should learn the fundamental concepts of measurement though concrete experiences. They should be able to measure distance, mass (weight), time, capacity, temperature, and angles. They should learn to calculate simple perimeters, areas, and volumes. They should be able to perform measurement in both metric and customary systems using the appropriate tools and levels of precision.

10. **Geometry:** Students should understand the geometric concepts necessary to function effectively in the three-dimensional world. They should have knowledge of concepts such as parallelism, perpendicularity, congruence, similarity, and symmetry. Students should know properties of simple plane and solid geometric figures. Students should visualize and verbalize how objects move in the world around them using terms such as slides, flips, and turns. Geometric concepts should be explored in settings that involve problem solving and measurement.

11. **Statistics:** Students should plan and carry out the collection and organization of data to answer questions in their everyday lives. Students should know how to construct, read, and draw conclusions from simple tables, maps, charts, and graphs. They should be able to present information about numerical data such as measure of central tendency (mean, median, mode) and measures of dispersion (range, deviation). Students should recognize the basic uses and misuses of statistical representation and inference.

12. **Probability:** Students should understand elementary notions of probability to determine the likelihood of future events. They should identify situations where immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make predictions such as election results, business forecasts, and outcomes of sporting events. They should learn how probability applies to research results and to the decision-making process.

Source: National Council of Supervisors of Mathematics. 1989. "Essential Mathematics for the 21st Century." *Arithmetic Teacher* (September 1989): 44-46.

A Manipulatives Table

Kostelnik, Soderman, and Whiren (1999) offer the following guidelines for setting up a manipulative materials table or center.

1. Place a large table with chairs adjacent to open shelves in any convenient location.
2. Provide ample materials at varying levels of difficulty on the shelves, well spaced for younger children. Cluster similar materials together. Materials for all aspects of mathematics and quantitative thinking should be available.
3. Provide clear plastic containers labeled with words or pictures for cleanup.
4. Introduce new games, materials, and tasks; demonstrate the use and care of the materials.
5. Provide a balance of open-ended materials (pegs, Legos, sewing cards), self-correcting materials (wooden cylinders, puzzles, nesting boxes), collectibles (bottle caps, buttons, sea shells, bread tags), and games (lotto, concentration, and cards).
6. Provide writing materials for older children to record their work, or place the center near the writing center.
7. Refrain from storing all manipulatives on the display shelves. Rotate items from the storage area to the display area regularly.
8. To keep interest high, rotate materials between mornings and afternoons for children in full-day programs.
9. Display written directions or pictographs to indicate what to do or how to do an activity. Also, display written materials, such as directions for games or the use of manipulatives and related books, so children can see them.

use in mathematics activities. Children should use manipulatives to represent mathematics in concrete form, and then proceed to using pictorial models and abstract symbols. There should be items for drawing and writing about mathematics during problem solving, as well as geometric shapes and other materials for building and exploring.

Some materials commonly included in manipulative kits are unifix cubes, pattern blocks, two-sided counters, geoboards with bands, play money, number cubes, color tiles, hundreds boards, base ten blocks, a Judy clock, place value mats, tangrams, geoblocks, fraction bars, and fraction circles. Household items such as bottle caps, bread tags, buttons, and keys are also often easily available. The exact composition of each kit will vary depending on the grade level for which it is designed.

Use of Calculators and Technology

Another important tool for the mathematics classroom is technology. As with manipulatives, the way a teacher uses these tools determines their effectiveness for learning mathematical ideas. The computer environment provides many visual representations—graphs, diagrams, geometric figures, and moving images. The sheer graphic power of technological tools allows children to explore and manipulate many different visual representations of mathematical models.

Virtual manipulatives are now available for investigating the characteristics of geometric shapes (Moyer, Bolyard, and Spikell 2002). A child as young as kindergarten age can explore complex repeating patterns using virtual manipulatives on the computer screen. Virtual manipulatives are available free on the Internet and can be found at the National Library of Virtual Manipulatives (http://matti.usu.edu/nlvm/nav/vlibrary.html) or in the electronic examples on the NCTM Web site (www.nctm.org). There are also many computer software programs specifically designed to support children's development of problem solving and procedural skills that are available for purchase. Because technology use is such an important part of literacy (both language and mathematical), children need many varied opportunities to learn to use technology effectively as a tool for their learning.

The calculator is also an effective tool for use in the mathematics classroom. Calculators have become standard materials in many schools because of their ability to expand children's opportunities for using different representations to explore mathematics.

The use of the calculator for mathematics instruction in the early grades has caused great debate within the mathematics community and among the general public (Battista 1994; Calvert 1999; Goldenberg 2000; Mercer 1992). Calculator use can have an effect not only on *how* children learn but also *what* they learn. As with many other mathematics tools, it is not the tool itself that causes the debate but how it is used for teaching and learning. Advocates for calculator use in the early grades propose that children use calcula-

tors in school mathematics to explore patterns and relationships with numbers. For example, young children can explore patterns among very large numbers with the support of a calculator or create a graph of a large data set using graphing programs.

When tedious computations are involved, most adults use more efficient methods, such as estimation or calculators, and we should teach children when and how to use these strategies, too. Teachers should guide children in recognizing when different strategies, such as using a calculator or using mental computation, are most appropriate.

This does not mean that children should use calculators when they are working to develop computational fluency. However, during problem solving or when working with large numbers, calculators can be used to relieve the burden of complex computations and to help students focus on the concepts being presented. Opportunities to use calculators as a part of the curriculum throughout the year help children to recognize when calculator use is appropriate.

Good Teaching Is the Key

Of course, effective mathematics instruction begins with effective teaching. No lesson, no matter how well planned, can be successful if the elements of effective teaching are not in place. For this reason, principals and teachers need an understanding of good teaching practices.

Effective teachers of mathematics must be effective teachers overall, meaning that they exhibit effective classroom management skills, actively engage their students, and make efficient use of instructional time. A mathematics lesson cannot succeed if the other elements of teaching such as classroom management, a logical progression of lessons, and time management are not in place. Jarrett (1997) explains that the classroom should allow

> students significant freedom to create, chart their own learning, debate, and engage in activities, their explorations should be within a structure. The teacher provides this structure with management strategies that help to create a safe, well-organized, and effective environment where all students can learn. She orchestrates discussions so that student participation and thinking are at a high level. She also ensures that students understand the core content in every lesson…[and] she maintains control-at-a-distance over the entire class, and actively monitors student behavior by moving around the room and speaking with individual students (online).

In addition to incorporating the general elements of effective teaching, a good

mathematics lesson must be based on a thorough understanding of how children learn mathematics and how the essential mathematics skills can be best taught. The effective mathematics teacher must prepare well-developed, developmentally appropriate lessons within an effectively managed classroom environment. When observing a mathematics lesson, a principal needs to look for evidence of teacher competence in both of these areas.

Several themes emerge from the extensive research of effective teachers and productive classrooms. First, good instructional management is key. Kilpatrick et al. echo this idea, asserting that "the quality of instruction is a function of teachers' knowledge and use of mathematical content, teachers' attention to and handling of students, and students' engagement in and use of mathematical tasks" (2001, 315). Wilson (1996) also highlights the important relationship between management and instruction: "The job of the teacher is first and foremost to instruct, not to manage. Yet management and instruction are inherently interdependent—in order for the learning environment to be at its best, both elements must be present, and working side by side, all the time" (2).

Second, actively focusing on students—and providing an environment in which they actively focus on learning—is vital. Mary Beth Blegen, teacher-in-residence at the U.S. Department of Education, puts a very personal face on what this means in practice:

> My day as a teacher was filled with all kinds of kids. What I taught wasn't ever as important as what was happening for those kids in that room. A yellowed piece of paper taped to my desk asked me every day, "What are the kids taking from this class into their world?" If I listened to that question, the classroom was a place of questioning and arguing and thinking with kids at the center. If I forgot that question, the classroom too easily became my classroom with kids only filling in the spaces" (2000, 12).

Third, effective teachers make excellent use of time. They have clear goals and align curriculum and instruction to achieving these goals.

As you observe teachers' mathematics lessons, look for these components of effective teaching. In your role as principal, your assistance is key in helping your staff members become good teachers in general, as well as effective teachers of mathematics.

Notes, Reminders and Ideas:

Section 4:

Using Assessment Data to Inform Mathematics Instruction

Assessment is a necessary component of instruction and a vital tool in any effective mathematics program. Good assessments provide data that help teachers determine the current knowledge and skills that individual students possess as well as provide feedback on the instructional program. When analyzed and interpreted, the information generated by assessments can be used as the basis for important instructional decisions about pacing, remediation, and the introduction of new information, as well as staff development and program design. The National Council of Teachers of Mathematics explains this relationship clearly:

> Assessment should be more than merely a test at the end of instruction to see how students perform under special conditions; rather, it should be an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done *to* students, it should be done *for* students, to guide and enhance their learning (2001, online).

For such an assessment system to be effective, our understanding of assessment must move away from reliance on a single test toward a system based on evidence from multiple sources, including teachers' professional judgments (NCTM 1995).

There are a wide variety of formative and summative assessment methods available for measuring student progress in mathematics, including observations, interviews, conferences, portfolio collections of work samples, open-ended questions, constructed-response tasks, selected-response items, performance tasks, journal writing, classroom discussions, and daily seatwork and homework assignments. Given the nature of these tasks, the assessment itself may look like any other classroom activity.

Major Shifts in Assessment Practices	
Toward	**Away From**
Assessing students' full mathematical power	Assessing only students' knowledge of specific facts
Comparing students' performance with established criteria	Comparing students' performance with that of other students
Giving support to teachers and credence to their informed judgement	Designing "teacher-proof" assessment systems
Making the assessment process public, participatory, and dynamic	Making the assessment process secret, exclusive, and fixed
Giving students multiple opportunities to demonstrate their full mathematical power	Restricting students to a single way of demonstrating their mathematical knowledge
Developing a shared vision of what to assess and how to do it	Developing assessment by oneself
Using assessment results to ensure that all students have the opportunity to achieve their potential	Using assessment to filter and select students out of the opportunities to learn mathematics
Aligning assessment with curriculum and instruction	Treating assessment as independent of curriculum or instruction
Basing inferences on multiple sources of evidence	Basing inferences on restricted or single sources of evidence
Viewing students as active participants in the assessment process	Viewing students as the object of assessment
Regarding assessment as continual and recursive	Regarding assessment as sporadic and conclusive
Holding all concerned with mathematics learning accountable for assessment results	Holding only a few accountable for assessment results
(Source: NCTM 1995)	

Because mathematics assessment is key to ensuring that instruction is both on-target and effective, a school's assessment activities should be a carefully planned component of its instructional framework. By using carefully developed and selected assessments to monitor and evaluate students' progress, schools can ensure that the mathematics instruction provided is consistent with student needs.

NCTM's 1995 *Assessment Standards for School Mathematics* identify the following features of a good mathematics assessment system:

- Student assessment is aligned with, and integral to, instruction
- Multiple sources of assessment information are used
- Assessment methods are appropriate for their purposes
- All aspects of mathematical knowledge and its connections are assessed
- Instruction and curriculum are considered equally in judging the quality of a program.

Purposes of Mathematics Assessment

Generally speaking, the purposes of mathematics assessment can be divided into four broad categories that complement and reinforce one another in a cyclical manner. Figure 3 on the next page, developed by the National Council of Teachers of Mathematics, illustrates the relationship that exists between each of the four and the actions that result from the use of assessment data in conjunction with each.

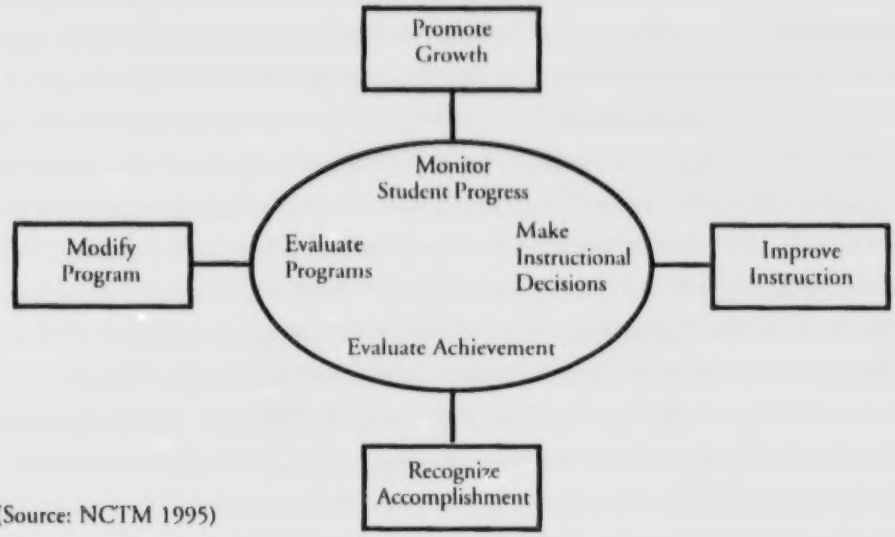
Evaluate Achievement

To begin with, teachers can use assessment measures to gain insight into students' progress toward achieving learning goals. Performance goals should be set for individual students on the basis of their current achievement levels, and evidence should be collected to provide both the teacher and the student with feedback about the student's progress toward these goals. This feedback should then be used to inform instruction and to revise learning goals, if necessary. The basic question to be answered when assessment is used for this purpose is, How is each student progressing in relation to the goals we have set and agreed on?

Make Instructional Decisions

After collecting information about student progress, teachers should analyze it to determine areas of strength and weakness and to plan further instruction. For example, if an interview with a child reveals that he or she doesn't understand place value, the teacher should then engage the child in activities that reinforce the concept of place value. The teacher should always keep in mind the basic question, How can I use evidence about my students' progress to make instructional decisions?

Figure 3: Purposes of Mathematics Assessment



(Source: NCTM 1995)

Monitor Student Progress

To evaluate student progress toward goals and to provide optimal instruction at the targeted level, teachers should assess student progress at regular intervals, both formally and informally. Even though children develop mathematical skills at varying rates, teachers should set target dates for skill acquisition to ensure that all topics are taught and mastered by the end of the school year. Also, periodic assessment reports provide parents and administrators, as well as teachers, with key information about student learning, the effectiveness of instructional programs, and the likelihood that student achievement goals will be met. When using assessments for this purpose, the basic question to answer is, How does each student's understanding at this time compare with the goals he or she was expected to achieve?

To determine how well an assessment reflects the mathematics that students need to know and be able to do, ask such questions as:

- What mathematics is reflected in the assessment?
- How does the assessment engage students in realistic and worthwhile mathematical activities?
- How does the assessment elicit the use of mathematics that is important to know and be able to do?
- How does the assessment fit within a framework of mathematics to be assessed?

- What inferences about students' mathematical knowledge, understanding, thinking processes, and dispositions can be made from the assessment (NCTM 1995)?

Evaluate Programs

Finally, to complete the cycle, assessments must also be used to evaluate programs. By providing evidence of student performance, assessments allow educators to draw conclusions about program effectiveness. For instructional practices to lead to achievement gains, the program must be one that meets student needs and addresses diverse learning needs. A careful analysis of assessment results can help determine the degree to which the instructional program supports the school's goals for student learning and achievement. The question being addressed here is, How well is the mathematics program working in relation to goals and expectations for the students (NCTM 1995)?

Questions to Ask

After collecting data from assessments, several questions should be asked. In general, questions that ask "Which, What, Why, or Where?" are better than those that ask "How many?" For example, asking which students are not meeting the standards in mathematics is better than asking how many students are meeting the standards. Examples of useful questions may be:

- How often did the students demonstrate...?
- How did different children (or teams) differ on...?
- Where are the gaps? What was disappointing?
- What worked?
- What resources can I/we utilize to address the issues?
- Where do I/we need to change and improve?
- What strategies are most apt to create learning?
- Which strategies need to be abandoned as ineffective?

(Source: Excerpted from Jandris 2001, 32).

Ultimately, educators use assessment as a tool to "collect data to better understand the current knowledge and skills that individual students possess" (Jandris 2001, 3). Assessment practices are most useful and informative when viewed as the shared responsibility of all individuals who are concerned with students' mathematical achievement (NCTM 1995).

Teachers and principals who work together to discuss and analyze assessment data are taking an important step to continuously improving mathematics instruction schoolwide.

Guidelines for Assessing Mathematics Achievement

Use a Balance of Formal and Informal Assessment Methods

By understanding various evaluation methods and choosing assessments that match the purposes for testing, teachers and administrators increase the likelihood that the data needed for solid decisions will be available. In Valencia's view,

> A complete assessment system....includes a balance of formal normative tests that help teachers and administrators know how students are performing compared to other students across the nation or the state; formal assessments published in conjunction with instructional programs that help teachers and students know how well students are learning; informal classroom work samples, performances, and observations that help teachers and students evaluate the application of skills to everyday learning; and student self-assessment that helps students become self-directed learners (1997, online).

The term "formal assessment" often brings to mind visions of children sitting in straight rows filling in "bubbles" on a standardized achievement test. In some instances, depending on the grade level, these tests include multiple-choice items designed to measure a specific mathematical skill. For younger students, such tests may be administered orally, with students prompted to circle or point to a word or picture.

When the results are reported, scores frequently appear in the form of percentiles, grade equivalents, and national curve equivalents. Often these scores are then aggregated by grade and school to allow comparisons across individuals, classes, schools, and states (Stowell and Tierney 1995). Such tests as the California Achievement Test, the Iowa Achievement Test, and the Stanford 9 fall into this category of assessment.

Obviously, standardized formal assessment measures have some drawbacks and cannot be used to fulfill some of the many purposes for assessment discussed earlier. Perhaps the biggest drawback of such tests is that scores are received several weeks, if not months, after the test is taken—too late to be useful in planning instruction for the group of children who took the test. In addition, formal standardized tests often are given infrequently, may have questionable reliability given children's rapid developmental gains in the early years, and are time-consuming to administer (Stowell and Tierney 1995). Despite these drawbacks, standardized mathematics assessments do serve a purpose. They allow the overall effectiveness of programs to be assessed, and many allow students' scores to be compared across classrooms and schools.

Early Childhood Testing Guidelines

The National Association for the Education of Young Children and the National Association of Early Childhood Specialists in State Departments of Education developed guidelines to aid educators in making informed decisions about curriculum, content, and assessment; evaluating existing curriculum and current testing practices; and transitioning to more appropriate approaches. With regard to assessment, they suggest the following principles, among others:

- Curriculum and assessment are integrated throughout the program; assessment is congruent with and relevant to the goals, objectives, and content of the program.
- Assessment results in benefits to the child such as needed adjustments in the curriculum or more individualized instruction and improvements in the program.
- A regular process exists for periodic information sharing between teachers and parents about children's growth and development and performance. The method of reporting to parents does not rely on letter or numerical grades, but rather provides more meaningful, descriptive information in narrative form.
- Results of screening tests are not used to make decisions about entrance to school or as the single criterion for placement in a special program, but rather used as part of a thorough process of diagnosis designed to ensure that children receive the individual services they need.
- Performance data of children collected by teachers to plan instruction are summarized and quantified by teachers and administrators to use in evaluating how well the program is meeting its goals for children and families.

Source: National Association for the Education of Young Children and National Association of Early Childhood Specialists in State Departments of Education 1990, 32-34.

Embed Assessments in Instruction

The value of frequent informal assessment measures is becoming more widely recognized. As Valencia states, "In the past ten years, we have witnessed a revolution in assessment, one that has finally taken hold in classrooms, schools, districts, states, and the nation" (1997, online).

Informal assessment has actually always been part of the repertoire of good teachers. Such techniques as observations, interviews, inventory tests, short diagnostic tests, portfolios, and presentations provide teachers with immediate feedback that can be used to improve and adjust the instructional methods they use with individual children. In addition, these assessments can be embedded in regular classroom instruction. Teachers should look at the activities they conduct every day and ask themselves, What do these assessments tell me about the students in my class?

The current trend in mathematics curricula toward active, hands-on learning for understanding provides plenty of opportunities for informal authentic assessments. Rubrics can be used to grade projects and problem-solving, and the focus on cooperative learning provides many opportunities to conduct interviews with students and observe their work (Bay et al. 1999). Isaacs and Carroll stress that "assessment should help the teacher evaluate not only answers but also how students are getting those answers and whether students understand the underlying concepts and connections" (1999, 512). They continue,

> For example, a student may appear to know the basic facts during problem-solving activities but actually be relying on counting. Another student might be quite proficient on isolated facts but have a weak grasp of the concepts of operations. A combination of assessment techniques can clarify each student's strengths and weaknesses and can help the teacher plan instruction (1999, 512).

Student work samples. Student work samples serve as an excellent source of information about what students understand and are capable of doing. By looking at student work, it is possible to determine a student's level of conceptual understanding and whether or not the student is able to apply skills across a range of problem types. For example, work samples may reveal whether or not a student is able to solve not only simple addition problems but also addition problems that are embedded in word problems.

Observation. Much information can be gleaned by observing students as they are engaged in problem-solving activities and games. By continually moving about the room, pausing to listen to students' conversations, and observing the ways in which students use materials, teachers can gain insight into how students arrive at answers (Ross and Kurtz 1993). For example, as students browse in the classroom grocery store and make purchases, the teacher may notice that they are using counting strategies as they compute the total price of the goods they would like to buy or that they are experiencing difficulty determining whether they have enough money to buy the items they wish to purchase. Children's facial expressions, comments, and the speed with which they are able to reach solutions all reveal children's ability, understanding, and comfort level with mathematics skills.

Guidelines for Looking at Student Work

The following guidelines were developed by the Philadelphia Education Fund to help teacher teams get started with a process of inquiry. These guidelines were informed by the work of a variety of education organizations and practitioners.

1. Gather a team, or small group, of teachers together.
2. Select a piece of student work. The sample should demonstrate a rich variety of student learning. It can be a work-in-progress, a final piece, or a document of a performance. Also, collect the scoring guide or rubric used to assess that piece. Make copies for team members, if possible.
3. If someone in the group is not familiar with your unit of study, take a few minutes to introduce its overall purpose, the activities that have been conducted, and the work that has been generated.
4. Discuss and write down one standard from those that you expected students to address in this activity. What did you expect the students to know and be able to do?
5. Next, take a few minutes to look at the work as a group. Either read it aloud or let each person take a turn looking at it.
6. Write down the group's observations about the work. Then write down comments and questions. You might allow each team member to do this first individually and then share in turn.
7. Next, use your scoring guide or rubric to assess the piece of work. If you do not yet have a scoring guide, reread the standard you have identified and assess the work based on its criteria. (You might put together an informal rubric by doing this.)
8. Take a few minutes to discuss as a team the following questions: What can you see from your observations, comments, and questions that will help you assess student learning? How might these observations determine your next steps as a teacher? Do these observations tell you anything new about your unit of study or classroom activities?

Source: Philadelphia Education Fund undated (online).

By recording brief observational notes as they monitor students at work, teachers remember these important "clues" and can use them in planning individual and whole-class instruction. Such notes may look like this: *John is still counting on, even for easy facts; Sarah seems to know most of the double facts and is able to use this knowledge to solve most of the near doubles* (Isaacs and Carroll 1999).

Conferencing. Short conferences with individual students are an excellent way to get a full picture of a student's level of understanding, problem-solving processes, thought processes, and ability level. These informal interviews are best done as teachers are observing students at work. The interviews are often short and consist of the teacher asking questions to probe the student's level of understanding and thought process. The following transcript illustrates how a conference might proceed:

Student: [Student reads] Three plus five equals…Hm. [Pause] Three. [Student then counts on fingers, putting up five fingers at one time.] Four, five, six, seven, eight. Eight.

Teacher: How did you figure that out?

Student: I did it on my fingers.

[Child is shown card with 5+5 on it.]

Student: [Rapidly] Five plus five equals ten.

Teacher: How did you get that?

Student: I figured it out in my mind.

Teacher: You always knew it? [Student indicates yes.] Okay, what's six plus six?

Student: [Fairly rapidly] Thirteen.

Teacher: How did you get that?

Student: Because I counted five and then added two more: five plus five and two more.

Teacher: And you got what, thirteen?

Student: Yeah. [Child reads next problem] "Seven plus nine equals…." [Pause. Then child begins to count on fingers. First, child apparently begins to count all – to seven on one hand. Then starts over, saying seven and starting over on fingers, putting up nine fingers one at a time.] Eight, nine, ten…., sixteen. Sixteen.

Teacher: Sixteen. Okay, so here's another one. If seven plus nine is sixteen, what's nine plus seven?

Student: [Two seconds, then child responds with enthusiasm.] Sixteen!

Teacher: How do you know that?

Student: It doesn't matter which one is first. But they're always…they're always like…. no matter what is first, they're always the same number.

Teacher: Here's another one. Four plus ten.

Student: [Quickly] Four plus ten is fourteen.

Teacher: How did you get that?

Student: Well, I just figured it out.

Teacher: On your fingers?

Student: No, I thought (Isaacs and Carroll 1999, 513).

By asking such questions as "How did you know that?" and "How did you get that?", the teacher leads the student into verbalizing his or her thought process. Often, this information is more valuable than knowing whether or not the student got the right answer, because it reveals whether the student understands the concept or only made a lucky guess.

Alternatively, a teacher can also ask questions during large-group activities. For example, if a teacher is modeling the building of an A-B-C pattern with unifix cubes, he or she might ask a student what color cube should come next. The answer should be followed with a question, such as "How did you know?" or "Why does it need to be this color?" Other students will benefit from this exchange between teacher and student and from the opportunity to hear the student's explanation. Sample questions to use in conferencing are provided on the next page.

Rubrics. A rubric is also helpful to teachers as they assess student responses to "interview" questions, written responses, or other types of assessments. With a rubric to guide the assessment measure, teachers are more likely to apply the assessment consistently across students. In addition, explaining the rubric to students shows them how a task will be evaluated and what specific proficiencies they are expected to achieve. A sample rubric is provided on page 43. This rubric can be used to evaluate a student's work and response when completing a word problem.

Portfolios. Portfolios provide another means by which teachers can assess mathematical skills. Many teachers choose to use this assessment method, in conjunction with other formal and informal means, for the unique benefit portfolios provide—they allow evidence of student growth to be easily seen over a period of time (Stowell and Tierney 1995). This aspect of portfolio assessment appeals to teachers, administrators, and parents alike—and often provides the means for an in-depth conversation about a child's progress over the course of a semester or year.

A mathematics portfolio for a student contains a variety of pieces of work, collected over an extended period of time and representative of different skill areas. Often, teachers and students collaborate to choose the examples included in the portfolio. As they select work samples, the teacher and the student discuss the work, noting the student's strengths, weaknesses, and areas of improvement. Depending on the age of the student, he or she may write a paragraph describing why a particular assignment was included in the portfolio.

Question Prompts for Conferencing

Patterning: If I make a pattern, can you continue the pattern? Can you make a pattern for me to try to continue? Look at my pattern. Can you make it the same way? Can you do it the opposite of mine?

Classifying: Can you put these into groups so that the ones that are alike are together? Why did you put them into those piles? Can you do it another way?

Counting: How many do you have there? Show me how you count them. Can you think of a way to arrange them so that if you lost count, you would not have to start all over again? Which pile has more? Can you make this pile with one more? One less?

Calculating: What if I take away ten from this pile? Add twenty? How many will there be then? You said that five plus two was eight. Can you show me with these objects? What is one more than this? One less? How many dinosaurs do you have? Close your eyes. I just hid some of them in my hand. Can you tell me how many I am hiding? How did you know?

Ordering: Can you put them in order from shortest to longest? Close your eyes. If I take one away, can you put it back where it was before? Here is a new thing. Can you put it in with these things that you have already ordered?

Measuring: How tall (heavy, wide, etc.) is that? How can you find out? What is about the same size? Can you use this string to make a drawing that is exactly the same size as your tower? Do you think that this chain will make it all the way to my desk? How many more links will you have to add? Which one is bigger? How tall is it? Can you make this one the same size?

Estimating: Estimate how many are here in all. Next find out how close you are. Did you estimate high or low? Where you close? How close?

(Source: Heuser 2000, 292)

Sample Rubric

This sample rubric can be used to assess student responses to open-ended word problems that require students to show their work and explain the process through which they arrived at an answer.

Student demonstrates proficiency—Score Point = 3

The student provides a satisfactory response with explanations that are plausible, reasonably clear, and reasonably correct, e.g., includes appropriate diagram(s), uses appropriate symbols or language to communicate effectively, exhibits an understanding of the mathematics of the problem, uses appropriate process and/or descriptions to answer the question, and presents sensible supporting arguments. Any flaws in the response are minor.

Student demonstrates minimal proficiency—Score Point = 2

The student provides a nearly satisfactory response which contains some flaws, e.g., begins to answer the question correctly but fails to answer all of its parts or omits appropriate explanation, draws diagram(s) with minor flaws, makes some errors in computation, misuses mathematical language, or uses inappropriate strategies to answer the question.

Student demonstrates a lack of proficiency—Score Point = 1

The student provides a less than satisfactory response that only begins to answer the question, but fails to answer it completely, e.g., provides little or no appropriate explanation, draws diagram(s) which are unclear, exhibits little or no understanding of the question being asked, or makes major computational errors.

Student demonstrates no proficiency—Score Point = 0

The student provides an unsatisfactory response that answers the question inappropriately, e.g., uses algorithms which do not reflect any understanding of the question, makes drawings which are inappropriate to the question, provides a copy of the question without an appropriate answer, fails to provide any information which is appropriate to the question, or fails to answer the question.

(Source: Kantrov 2000)

A typical mathematics portfolio might contain such work samples as solutions to difficult problems that detail problem-solving abilities, work that shows the use of mathematics in another discipline, original problems created by the student, examples of the student's group activity, written reports on a major topic in math, the student's written account of his or her growth in mathematics, and responses to challenging questions and problems (The Learning Network 2002).

Focusing on Assessment Data

As schools and districts begin to make more extensive use of mathematics assessment data, they should look first at the instruments they use to measure student learning. Fox suggests that an analysis of the tests should include a look forward to the data they would provide. For example, to what degree does the mathematics assessment "provide data that lead to purposeful, targeted and systematic instruction?" (2000, 22). The best assessments have been carefully aligned with standards so they provide information about knowledge and skills that students are expected to master.

However, even if the test is a good one, the results of the assessments may not be helpful if they are not presented in a user-friendly format. The Mesa Public School District in Arizona ensures that each school *does* have helpful, on-target data available to it by annually providing each principal with a data book that presents the school's achievement indicators from the previous year (e.g., Stanford 9 scores, district test results, AIMS results, etc.). The data are organized and analyzed to enable the school staff to answer the following questions:

- Are our overall scores acceptable?
- How did students do on specific objectives?
- How did different sub-populations score? (analyzed by ethnicity, mobility, poverty, and language-minority status)
- How have scores changed over time?

School staff are then expected to be "educational detectives [who] develop hypotheses about the reasons for strengths and weaknesses revealed by the data, and [who] are asked to 'Look at the patterns, see what areas you can improve, and develop improvement strategies'" (Educational Research Service 2000, 2).

Schools that have embedded use of assessment data into their instructional improvement efforts have also found that using the results of only one annual test is too limiting. Specifically, such results "provide too little information too infrequently to allow teachers to adjust their instruction to reflect changing student needs during the course of the academic year" (Research and Policy Committee of the Committee for Economic

Development 2001, 15). Thus, many districts develop additional assessments—typically much shorter assessments that can be given more frequently—that parallel the state-required assessments.

Pam Davis, an assistant principal in a Brazosport Independent School District middle school, explains how data from assessments developed by teacher teams are used in her school:

> We develop a profile sheet for every child using the data from the periodic assessments—and kids as well as teachers are able to use them. What we're looking for is a pattern over several assessments that a particular objective hasn't been mastered. Every day children who need them are provided with tutorials—with kids grouped by objective.... It takes organization to group and regroup kids and teachers on a daily basis—but it's been worth it (Cawelti and Protheroe 2001, 30).

In addition to providing information about what specific students have—or have not—mastered, the results have helped teachers identify their own personal strengths and areas in which they would like to improve. An important additional benefit of the use of assessment data has been the encouragement it provides for teachers to discuss and share instructional strategies.

Important First Steps

In an interview conducted by Dennis Sparks, Mike Schmoker provides some concrete suggestions for a school or a district that wants to begin using assessment data to improve instruction:

> The school should concentrate on three very simple things—focused, collaborative learning, measurable goals, and data. Begin by looking at the data to establish one or two measurable goals. Have teachers get together in the summer when they have time and can be more relaxed to select or create periodic assessments by which they will measure progress related to that goal. Then have teams of teachers get together regularly to talk about their progress, focusing like a laser beam on specific emergent problems preventing students from doing well relative to that year-end goal. If a school does these things, it is all but certain to make some real progress (Sparks 2000).

With respect to mathematics instruction, this translates into providing teachers with adequate time to develop and familiarize themselves with available mathematics assessments and materials, creating a common planning period or block of time during the week

for teachers to meet and review the progress of their students, and establishing target goals for students' mathematics achievement over the course of the year.

Making instructional improvements in mathematics teaching and learning that are documented through varied assessments of student achievement will not take place overnight. Too often when it comes to mathematics achievement, some people look for a quick fix. Targeting measurable goals and achieving those goals will be a slow, systematic process that requires focusing on one or two important mathematics objectives each year with ongoing support from the school principal. Mathematics teachers will need to participate in high-quality staff development on a range of topics including, using data for decision-making, setting student goals, using instructional techniques, and effectively using a variety of formal and informal assessment methods (Bay, et al.,1999).

As Jandris (2001) makes clear, teachers and administrators must have an "assessment toolbox" that is full of balanced and high-quality assessment instruments. The tools must include a variety of assessment instruments—both standardized tests and classroom-based assessments—as well as the knowledge of how to effectively use and interpret the data collected. Using a variety of assessment techniques yields a rich set of data that may provide insights into student progress and the effectiveness of certain instructional styles. As the school leader, it is your responsibility to see that your teachers possess a well-stocked toolbox of assessment methods and to help them use these tools to modify and improve the learning that takes place within their classrooms.

Notes, Reminders and Ideas:

Section 5:

Providing Extra Support for Students Struggling with Mathematics

Even with high-quality mathematics instruction, some students may fall behind. These students often become confused, anxious, and "turned off" to math. This creates challenges for teachers who are responsible for designing and implementing an instructional program that meets the needs of a class of students with a range of mathematical abilities.

The bottom line is this: Although good instruction can prevent most mathematics difficulties, there will always be students who need extra help to succeed.

Identifying Students Struggling in Mathematics

Many warning signs may signal a student who is falling behind in mathematics skills. Teachers can use lists of age-appropriate competencies to identify students who cannot demonstrate some of the skills typical for their grade level. As an example, using the partial list of fourth-grade skills presented earlier, a teacher might notice that a fourth-grader demonstrates little knowledge of the relationship between addition and multiplication. Additionally, King and Parker (undated) identify the following signals that students may be struggling in mathematics:

- Difficulty seeing and hearing numbers correctly, as evidenced by reversals and substitutions
- Lack of number sense or a lack of understanding about what numbers represent
- Failure to perceive patterns and relationships

- Energy concentrated on motor problems of forming the numerals and keeping place
- Lack of memory for number facts, mathematical vocabulary, or problem-solving procedures
- Loss of the train of thought in the problem solving process, especially when working with word problems
- Confusion when dealing with multiple mathematical concepts
- Insistence on using one solution or one method for all problems
- Possessing a different learning modality from that used to teach mathematics concepts.

King and Parker have found that computational errors or trouble with problem solving form the basis of mathematical errors 80 to 90 percent of the time. Knowing this, educators can take steps to reduce these errors by providing instruction designed to increase students' rapid recall of basic facts, understanding of processes, and comfort level with the material.

Exemplary and Promising Programs

The U.S. Department of Education has produced a booklet entitled *Exemplary and Promising Mathematics Programs.* Designed as a resource for principals, superintendents, curriculum specialists, and others directly involved in curriculum development, the booklet provides descriptions of five mathematics programs that have been deemed "exemplary" and five that have earned a "promising" status, with some designed for elementary and middle grades use. This is a valuable reference for those involved in developing, planning, and implementing mathematics curricular programs. (Available at www.enc.org/professional/federalresources/exemplary/promising/)

Differentiating Instruction to Provide Needed Support

Many adaptations and accommodations can be made to assist students who are struggling in mathematics (Ebeling, Deschenes, and Sprague 1994). Some of these are related to general instructional techniques that recognize the varying abilities and learning styles of students. Differentiated instruction allows students to work on material at their own level and provides much needed support for students who are struggling to progress. The following sections describe several instructional techniques that are considered "good

practice" and that allow teachers to differentiate instruction and provide additional support to meet the needs of students who are struggling in mathematics.

Questioning and Communication

Encouraging students to "think aloud" and share the ways they approach problems—a strategy mentioned earlier as a characteristic of good mathematics instruction—may be especially important for children who are having difficulty. NCTM (2000) suggests that teachers should "use communication to foster understanding of mathematics." This can be accomplished by encouraging children to use pictures, words, and symbols to communicate their mathematical ideas. A group of teachers offer these specific strategies for encouraging communication in mathematics:

- Ask students to make conjectures about "what might happen if..." situations. Return to those conjectures after students have investigated a problem to discuss which ideas still hold true.
- As students share their solutions with the class, have them describe the strategies they used to reach their solutions.
- Allow other students to ask questions of each student who shares his or her solution and method.
- Ask one student to rephrase or repeat another student's explanation (Kline 2000, 570).

Encouraging children to verbalize their thought process and to ask questions of others can be a very effective means of assisting children in developing mathematical understanding. As a child attempts to explain his or her thinking to a friend or the teacher, the child is able to gain a better understanding of the process he or she is trying to describe. As Kline writes, "sharing their strategies with classmates solidifies children's understanding" (2000, 570). Struggling students also benefit greatly from asking other students questions of clarification. Sometimes a student is able to explain a concept or a process to another student better than the teacher is able to because the students are operating on similar developmental levels.

The phrasing of questions is also important. Good questions are engaging and elicit conversation. The following episode, as described by a classroom teacher, illustrates the different responses obtained from children when the same question is posed in two different ways. The students are working in pairs to create a repeating pattern of green-yellow-yellow with unfix cubes. They have strung together nine cubes and the teacher has just asked them what color comes next. The teacher writes,

> They've done the pattern, but they couldn't tell me what comes next. And then I had to stop and think, "Okay. Now what do I say?" So then I have them start from the beginning and say a color as they point

> to each cube. And then they can tell me what comes next. But then I wondered if they were just repeating this string of words and not really recognizing their pattern. So with the next pair of students, I was prepared. I thought about what I could ask to get them to think about their pattern. I asked them, "Can you describe your pattern to me?" I was amazed at how differently students responded to this question. They talked about what the repeating unit was, how many pieces they needed to see in their pattern to figure out what the unit was, and how it was different from the pattern their partner had made (Kline 2000, 570).

In general, wondering about children's knowledge leads teachers to ask better questions, which, in turn, assists students in clarifying their ideas and gaining an understanding of a concept. Before asking a student a question, teachers should ask themselves this question: "Do I already know what I want the answer to be, or am I really curious about what this child is thinking?" (Richardson 1997).

Scaffolding Instruction

Good mathematics instruction progresses from the concrete level to the pictorial level to the abstract level during instruction as students develop an understanding of the mathematical skill or procedure being taught. Scaffolded instruction helps the learner to transition between these levels by first having the teacher demonstrate application of a new concept, then by supporting student application and practice, and finally, by providing opportunities for the student to solve problems independently.

Multiple Intelligence Theory

Over the past decade, educators have begun to explore the application of Howard Gardner's multiple intelligence theory to classroom instruction. For some students, the variation in instructional approaches that include musical and kinesthetic activities has been especially beneficial.

Students who struggle with mathematics may experience difficulty because they are unable to "see" the larger picture, to make associations, or to remember basic facts. Activities that incorporate the opportunity to act out a problem, sing a song, work cooperatively, or create a representation cater to these students' multiple intelligences and deemphasize learning all mathematics through logical, linear activities.

Some of these intelligences can be emphasized in mathematics activities in the following manner:

- Spatial: using visual clues
- Linguistic: Reading word problems

- Logical-mathematical: Creating and solving equations
- Kinesthetic: Exploring tactile models
- Musical: Creating auditory patterns
- Interpersonal: Sharing strategies
- Intrapersonal: Journal writing (King and Parker undated).

An elementary school in Maryland has introduced the theory of multiple intelligences into its curriculum with great success. A teacher writes, "In the five years since we began putting Gardner's theory into practice, the overall achievement and confidence of our students have risen substantially. The results have far exceeded our expectations" (Greenhawk 1997, 62). A key piece of this approach is helping students understand how to use their strengths and how to work on their weaknesses.

Popular Instructional Materials

Students who are experiencing difficulty performing mathematical operations often benefit from the use of materials designed to address visual and spatial difficulties. Below are some popular materials that are used to assist students experiencing difficulty in two areas: counting and operations (e.g., addition, subtraction, multiplication, and division) (King and Parker undated).

Operations	Counting
· Number lines	· Grasping aids
· Counting frame	· Velcro-sensitive mitt
· Eye-gaze frame	· Magnetic tape
· Large-print calculator	· Counting frame
· Talking calculator	· Number lines

Metacognitive Strategies

Metacognitive strategies assist students in knowing when, how, and why to use the skills and knowledge they possess. A middle school student employing metacognitive strategies looks at an algebra problem and asks, "What is it really asking me? What mathematical operations would I need to solve this? What step should I do first?" (Riggs and Gil-Garcia 2001, 24).

Redefining Success for Teachers and Students

Too often, we view success in mathematics in terms of the number of correct answers obtained by students. For all students, especially those who are struggling, this can be discouraging. With the new emphasis on the relationship between mathematics understanding and procedural fluency, schools have developed new expectations for both students and teachers that recognize the many steps involved in the problem-solving process and the teacher's role in facilitating success.

New Expectations for Students	Teacher Actions Consistent with Expectations	Classroom-Based Indicators of Success
Most "real" tasks take time to solve; frustration may occur; perseverance in the face of initial difficulty is important.	Use "good" tasks; explicitly encourage students to persevere; find ways to support students without removing all the challenges in a task.	Students were engaged in the tasks and did not give up too easily. The teacher supported students when they "got stuck" but did so in a way that kept the task at a high level.
Correct solutions are important, but so is being able to explain how you thought about and solved the task.	Ask students to explain how they solved a task. Make sure that the quality of the explanations is valued equally as part of the final solution.	Students were able to explain how they solved a task.
Students have a responsibility and an obligation to make sense of mathematics by asking questions when they do not understand and by being able to explain and justify their solutions and solution paths when they do understand.	Give students the responsibility for asking questions when they do not understand, and have students determine the validity and appropriateness of strategies and solutions.	With encouragement, students questioned their peers and provided mathematical justifications for their reasoning.
Diagrams, sketches, and hands-on materials are important tools for students to use in making sense of tasks.	Give students access to tools that will support their thinking processes.	Students were able to use tools to solve tasks that they could not solve without them.
Communicating with others about your thinking during a task makes it possible for others to help you make progress on the task.	Ask students to explain their thinking, and ask questions that are based on students' reasoning, as opposed to how the teacher is thinking about the task.	Students explained their thinking about a task to their peers and the teacher. The teacher asked probing questions based on the student's thinking.

Source: Smith, Margaret Schwan. 2000. "Redefining Success in Mathematics Teaching and Learning." *Mathematics Teaching in the Middle School* (February 2000): 382

Effective learners often engage in this process automatically, knowing that these key questions must be answered before they proceed in solving a problem; ineffective learners often do not. Thus, the teaching of metacognitive skills can lead to higher levels of learning (Riggs and Gil-Garcia 2001).

Research suggests that metacognitive strategies improve learning because they "provide [students] with an efficient way to acquire, store, and retrieve information and skills" (Allsopp and Kyger 1999). Many students who are struggling in mathematics experience difficulty because of an inability to recall information they previously knew. In this case, mnemonics, a strategy in which a meaningful word is developed from the letters in the word or phrase to be remembered, can be especially useful in triggering the retrieval process. For example, to remember the order of operations, students may learn the mnemonic phrase, "Please Excuse My Dear Aunt Sally" (Parentheses, Exponents, Multiplication/Division, Addition/Subtraction). Additional examples of mathematics mnemonics are included in Appendix B.

Students should be taught how and when to use metacognitive strategies, given ample practice time, and be provided with help in learning how to generalize the strategies to new problems and situations. Teacher modeling is the most effective way to ensure that students understand the purpose of a strategy and the circumstances in which it should be used. Some teachers choose to post visual reminders of strategies on classroom walls or to have their students keep journals in which they record the strategies that they have learned (Allsopp and Kyger 1999).

Specific Strategies to Support Struggling Mathematics Students

In addition to the general "good teaching" strategies just discussed, it may be necessary to implement targeted strategies to assist students struggling with particular mathematics concepts. King and Parker (undated) suggest the following:

- Stay with hands-on learning until understanding is achieved.
- Whenever possible, allow students to work together.
- Demonstrate how math is present in all subjects.
- Show students how a completed problem or task will look to give them the big picture.
- Teach estimation and have students use it.
- Have students set their own goals in mathematics.
- Have students solve several problems using one strategy and one problem using several strategies.

- Use real-world math wherever possible.
- Every math lesson should include a review of something already mastered.

There are many mathematical areas in which students commonly encounter difficulty; however, two of the most common problem areas for elementary school children are basic operations and problem solving. The following sections describe the nature of these difficulties in more detail and provide several strategies for remediating each problem.

Strategies to Assist Students Experiencing Difficulty with Basic Operations

Mastery of basic operations is vital to math success, as is the ability to form and remember associations, understand basic relationships, and make generalizations (Mercer and Miller undated). When teaching operations such as addition, subtraction, multiplication, or division, teachers should begin by developing understanding of the meanings of the operations before describing and modeling the procedure, and then give students plenty of opportunity to engage in both guided and independent practice. Students who need more support should remain in the guided instruction phase until they can work independently (Mercer and Miller undated).

An instructional sequence that progresses from the concrete level to the representational (pictorial) level to the abstract level is also recommended as a way to help struggling students develop a better understanding of mathematical procedures (Mercer and Miller undated). For example, a lesson on division that follows this sequence might begin with the teacher modeling division with twelve unifix cubes and dividing these into three groups of four to represent the problem "twelve divided by three." As he or she divides the groups, the teacher should explain that to make three groups from the twelve cubes, four cubes must be placed in each group. After demonstrating several problems in this manner, the teacher might then call on several students to solve simple division problems using the cubes. Next, students might be given their own set of cubes and be asked to solve a series of division problems at their desks or to create their own division problems with the cubes. Once students seem to understand the concept, they might be asked to draw pictures of their solutions, to write the corresponding number sentences, and finally to solve and explain a series of division problems without any prompts.

Allsopp and Kyger (1999) provide the following suggestions for assisting students who are experiencing difficulty with basic operations:

- Provide an example of a correctly solved problem at the beginning of every lesson.
- Have students verbally or visually explain how to solve a problem.
- Introduce only one concept at a time and teach to mastery.
- Teach in small chunks so students get lots of practice with one step at a time.

- Provide learning aids, such as calculators, to help students focus on conceptual understanding.
- Use estimation throughout the day and have students estimate a reasonable solution before starting any computation.
- Teach facts in families.
- Demonstrate all concepts with manipulatives.

Strategies to Assist Students Experiencing Difficulty with Problem Solving

Many students experience difficulty with problem solving (Jarrett 1999). For some students, the problem lies with an inability to read and comprehend the problem (Bley and Thornton 1995); others have trouble distinguishing relevant from nonrelevant information. Some students have trouble simply understanding what the problem is asking them to do. Karen Scrapple, a special education teacher at an elementary school in Oregon, suggests that the best way to assist these students is to help them break the process down into smaller steps. She writes,

> They can't seem to get it if it's too overwhelming. So we help them to break it down. And while they may not understand the core concepts at the same level as some of the [other] students in the class, they're doing the same process (of problem solving), and they're very excited about that (Jarrett 1999).

Problem solving requires students to read and analyze a problem situation and to then choose an appropriate strategy to obtain the answer to the problem. As Jarrett (1999) explains, "the student [uses] her prior knowledge of mathematics and reasoning skills to choose a strategy, apply it, and evaluate its effectiveness. The experience will [then] be added to the student's store of knowledge, to be called upon the next time she tackles a novel and challenging problem." Problem solving is an important skill that all students can master; a teacher's job is to equip students with an understanding of the necessary skills and strategies needed to confidently tackle problems of this type.

The following problem solving strategies can be effective for students of all abilities and all grade levels. As students progress, the strategies should be modified so students receive the proper amount of support.

- *Working backward.* This strategy is most effective when the answer or situation is known. The skill is most useful when such variables as cost, time, distance, and so on are known.
- *Simplifying and reducing.* This skill is useful in any situation involving numbers. Students sometimes become confused by large numbers; reducing the value of

the numbers used or rounding to the nearest whole number may assist the student in processing the problem, developing a representational picture, and ultimately solving the problem.

- *Recognizing patterns.* This strategy can be used in a variety of situations involving numbers, words, letters, shapes, and forms. Teachers can ask students to extend the pattern or state the rule for the pattern.
- *Drawing and modeling.* Drawing a picture or manipulating objects to form a model helps students visualize a situation, verbalize abstract ideas, and explain relationships.
- *Make a table or a graph.* Tables and graphs provide a means for students to organize and summarize numerical and verbal data. This may make it easier for students to see relationships and interpret data.
- *Act it out.* Some students learn best when they act out a problem kinesthetically. This often provides a concrete form of understanding. If acting out a problem is impractical, students can simulate the problem situation using manipulatives (Krulik and Rudnik 1996; Sorenson et al. 1996).

Teaching Problem Solving

Vaughn, Bos, and Schumm (1997) offer the following suggestions for teaching problem solving:

- Introduce students to problem solving early in the mathematics curriculum.
- Make problem solving the reason for computation.
- Provide assistance to students who are experiencing difficulty so that they can solve the problem. For example, if a student cannot read a problem, rewrite it using simpler words, or if a problem is too complex, break it down into smaller steps.
- Ask students to write their own problems and to develop variations of existing problems.
- Use familiar subject matter and contexts as the content for word problems.
- Monitor progress and modify problems as necessary (27).

Providing Supplementary Mathematics Programs

Sometimes students who struggle in mathematics need more support than the teacher can provide during regular instruction. One option is to place students in supplementary mathematics programs. These programs, which often must be purchased by schools, vary in nature, duration, focus, and cost. Before committing to a particular supplementary mathematics program, school staff should ask several questions to ensure that they choose the program that best meets the needs of the participants:

- Are the goals of this program consistent with our school goals for mathematics at this level?
- Can the program realistically accommodate the number of students we have in need of assistance?
- Must the program be taught by trained professionals? If so, how long is the training program?
- Are the methods of instruction used in the program compatible with the instructional methods used by the classroom teacher?
- What is the success rate of the program?
- What type of students stand to benefit most from this program? Does this match the description of our students most likely to be referred to the program?

Promoting a Positive Attitude toward Math

Above all else, schools should design mathematics instruction to ensure success and thus promote positive attitudes. Many students who experience difficulty in mathematics develop negative attitudes toward the subject and doubt their ability to succeed in the area. Attitudes, beliefs, and motivation have a large influence on one's self-confidence and perceived ability to succeed.

Mercer and Miller (undated) suggest the following guidelines for promoting positive attitudes toward mathematics:

- Involve students in setting challenging but attainable instructional goals. Goal-setting can exert a powerful influence on student involvement and effort.
- Encourage student success by basing instruction on previously learned skills.
- Use progress charts to provide students with feedback on how well they are doing.
- Discuss the relevance of a math skill to real-life problems. Use word problems that are part of a student's daily life.

- Communicate positive expectations for student learning. Students need to sense that their teachers believe in their ability to succeed in mathematics.
- Help students understand the idea that their own effort affects achievement outcomes. Constantly point out that what they do influences both their success and their failure.
- Model an enthusiastic and positive attitude toward math and maintain a lively pace during math instruction.
- Reinforce students for effort on math work.

Notes, Reminders and Ideas:

Section 6:

The Principal's Role in Observing and Evaluating Mathematics Instruction

As discussed earlier in this publication, effective mathematics instruction depends on good teaching. The most effective mathematics teachers not only are knowledgeable about instructional practices and the development of mathematics concepts but also are highly competent classroom managers. They hold high expectations for their students and have in place a series of procedures and routines for all classroom activities. Because students know what is expected of them and have been taught to work independently, the teacher has opportunities to work with small groups or individual students to provide each student with mathematics instruction tailored to his or her needs.

The teaching of mathematics is a complex process, complicated by the fact that students progress at different rates and learn best through different modalities. This creates a challenge for you, as the principal, when it comes to observing mathematics instruction and evaluating teacher performance. What should you look for when you observe mathematics instruction? The checklist on the following pages gives some guidance by listing what is required for effective mathematics instruction in each instructional strand.

Although most of these questions can be answered through observations, it may be necessary to make multiple observations to obtain a complete picture of the nature of mathematics instruction in a particular classroom. Teacher interviews can fill in "gaps" in information that may not be readily observable on any particular day. During these interviews, ask the teacher to explain his or her assessment system, including how "real data" is collected on each child. The teacher should also be able to describe the learning needs of each of the children in his or her classroom and to provide documentation of each child's performance and progress.

Guidelines for Observing Mathematics Instruction

Important Mathematics

- ✓ Does the **teacher** plan and deliver mathematics content consistent with district curriculum and assessment guidelines?
- ✓ Is the **student** engaged in mathematical tasks consistent with district curriculum guidelines?

Real-World Connections

- ✓ Does the **teacher:**
  - ❑ Routinely use data and graphs as problem-solving contexts?
  - ❑ Routinely provide examples of ways mathematics is used in the world?
  - ❑ Whenever possible, make historical connections?
  - ❑ Whenever possible, make connections to other subject areas?
- ✓ Can the **student** identify uses of mathematics in the real world and in other subject areas?

Problem Solving

- ✓ Does the **teacher:**
  - ❑ NOT present "rules" or "steps" for solving problems?
  - ❑ Develop and name problem-solving strategies?
  - ❑ Routinely use and name problem-solving strategies and processes in any kind of lesson?
  - ❑ Promote and model ways to monitor one's thinking?
  - ❑ Promote a disposition to formulate, represent, abstract, and generalize in problem situations?
- ✓ Can the **student:**
  - ❑ Name and use problem-solving strategies when solving problems?
  - ❑ Name and use problem-solving strategies in ALL mathematics contexts?
  - ❑ Name and use the processes of problem solving (i.e., understand, plan, solve, check)?
  - ❑ Monitor his or her thinking while solving problems?
  - ❑ Communicate his or her thinking and work while solving problems?
  - ❑ Find correct solutions or make appropriate decisions for problems?
  - ❑ Formulate, represent, abstract, and generalize in problem situations?

Exploring and Developing Concepts and Skills

- ✓ Does the **teacher:**
  - ❑ Whenever possible, introduce concepts and skills through problem solving or reasoning experiences that ask children to do higher-level thinking and to

communicate, rather than delaying problem solving or reasoning until children have mastered a procedure?

- ❑ When appropriate, use teacher-led classroom conversations to develop concepts and skills?
- ❑ Explain connections between math concepts/skills and other concepts/skills?
- ❑ When appropriate, use manipulatives to promote sense-making?
- ❑ Use and facilitate cooperative learning groups by listening and asking questions rather than telling?

✓ Can the **student:**

- ❑ Use sound mathematical reasoning when exploring concepts and skills?
- ❑ Communicate his or her reasoning when exploring concepts and skills?
- ❑ When appropriate, use manipulatives to make sense of mathematical situations?
- ❑ Make connections between manipulative models and symbolic representations?
- ❑ Explain connections between mathematical concepts/skills and other concepts and skills?
- ❑ Work cooperatively to explore concepts and skills and solve problems?

Number Sense

✓ Does the **teacher:**

- ❑ Emphasize the meaning of numbers and operations and the ways to represent numbers and operations?
- ❑ Give sufficient attention to developing numbers, operations, and number relationships before asking students to learn basic facts and algorithms?
- ❑ Emphasize that most calculations can be completed using prior knowledge and simpler calculations?
- ❑ Give children frequent opportunities to create estimation and mental math techniques?
- ❑ Help children understand computational strategies and use them appropriately and efficiently?
- ❑ Provide opportunities for children to demonstrate numerical reasoning not only during activities but also on assessment, practice, and homework?

✓ Can the **student:**

- ❑ Decompose numbers in different ways?
- ❑ Connect addition, subtraction, multiplication, and division with actions arising in real-world situations?
- ❑ Explain the effects of operating on numbers and how the operations are connected to each other?

- ❑ Choose, apply, and explain appropriate mental calculation techniques for finding exact answers and estimates?
- ❑ Correctly and efficiently perform calculations and discuss the underlying ideas?
- ❑ Determine whether the result of a calculation makes sense in the context of the numbers and real-world quantities involved?

Sense-Making

✓ Does the **teacher:**

- ❑ Not stop with the answer but get at the thinking behind it through the use of statements and questions such as "Explain" and "Why"?
- ❑ Solicit multiple approaches to the same correct answer?
- ❑ Help children develop autonomy by using logic, reasoning, and proof as verification rather than relying on the teacher for correct answers?
- ❑ Solicit and address children's "non-standard" or "limited" ideas?
- ❑ Emphasize learning to reason and constructing proofs as a part of understanding mathematics?

✓ Does the **student:**

- ❑ Use logic, reasoning, and proof as verification rather than relying on the teacher for correct answers?
- ❑ Expect mathematical concepts and procedures to make sense?
- ❑ Use reasoning and proof to make sense of mathematics?

Practice and Apply Concepts and Skills

✓ Does the **teacher:**

- ❑ Provide practice beyond skills to include reasoning, connections, communication, representations, and problem-solving?
- ❑ Make decisions about which children need what kind of and how much practice?
- ❑ Use a plan for frequent review?

✓ Can the **student** use concepts and skills correctly when reasoning, making connections, representing, communicating, and solving problems?

Balance Across Strands

✓ Does the **teacher:**

- ❑ Spend sufficient time exploring and developing the non-number topics in the program (e.g., geometry)?
- ❑ Make connections to other strands in the math curriculum?

✓ Does the **student** believe and can he or she explain how mathematics is more than arithmetic?

Guidelines for Observing Effective Mathematics Instruction

Technology

✓ Does the **teacher:**

- ❑ Use technology primarily to develop understanding?
- ❑ Help children choose an appropriate calculation technique?
- ❑ Allow the use of technology in problem-solving situations?

✓ Can the **student:**

- ❑ Choose an appropriate calculation technique in a given situation and explain the choice?
- ❑ Use technology to find correct answers and make appropriate decisions?

Assessment

✓ Does the **teacher:**

- ❑ Go beyond the assessment of skill proficiency and correct answers to assess understanding, reasoning, representation, connections, communication, and problem-solving?
- ❑ Assess during instruction through listening, watching, and questioning?
- ❑ Use multiple assessment techniques?
- ❑ Introduce test-taking strategies that emphasize reasoning?

✓ Does the **student** know and can the student use test-taking strategies in testing situations?

Beliefs

✓ Does the **teacher:**

- ❑ Promote the development of helpful attitudes and beliefs?
- ❑ Provide all students with challenging mathematics?
- ❑ Encourage and support all students?
- ❑ Recognize and reward all students for powerful thinking and reasoning?

✓ Does the **student:**

- ❑ Demonstrate helpful attitudes and beliefs about mathematics?
- ❑ Believe in his or her abilities to succeed in mathematics?

Source: Charles, Randall I. 1999. *What to Look for in a Powerful Mathematics Classroom.* Paper presented at the April 1999 Conference of the National Council of Supervisors of Mathematics in San Francisco.

During observations and visits to classrooms, take a moment to look around the classroom at the types of mathematics materials available to students. What types of manipulatives are available? Are there mathematics posters on the walls? Is student work in mathematics displayed? Are there books on mathematics in the classroom library? Is there evidence that mathematics is integrated into other content areas? In what type of activities are students engaged? A classroom visit should leave you with the feeling that students are immersed in a high-quality mathematics program that emphasizes hands-on learning, real-world connections, and conceptual understanding.

Working with Teachers to Improve Their Mathematics Instruction

The primary objective of teacher observation is to ensure that students are receiving high-quality, effective instruction in a supportive environment. Principals should be viewed as instructional leaders—individuals to whom teachers can turn for instructional guidance and support. In order to conduct effective observations that provide constructive feedback for the teacher and to fulfill their role as instructional leaders, principals must have an understanding of children's developmental progression of mathematics concepts as discussed in Section 3.

Principals who understand the process through which children develop mathematical understanding and the characteristics of effective mathematics instruction are well-equipped to conduct informed observations and provide constructive feedback and suggestions. During all conferences, it is important to remain supportive of the teacher's efforts and to provide justification for specific suggestions.

The following are suggestions for ways in which you, as the principal and the instructional leader, can work with teachers to improve their methods for teaching mathematics:

- Facilitate the development of a schedule that allows for peer observations. Encourage teachers at the same grade level to observe mathematics lessons in one another's classrooms. Teachers can learn a great deal from watching others teach and then discussing what they observed. Also, teachers who need support may find peer observations less threatening than administrator observations.
- Provide staff development offerings, such as workshops or seminars, in needed areas. For example, if you notice that many teachers cannot adequately monitor student progress, you might organize a staff development session on methods of mathematics assessment.
- Ask the mathematics curriculum specialist to observe a mathematics lesson in a teacher's classroom and provide constructive feedback and ideas for improvement. The specialist could also teach a lesson to a teacher's class while the teacher observes.

- Videotape a teacher teaching a mathematics lesson or series of lessons, and then watch and critique the videotape together.
- Provide a teacher with a list of resources, such as books or Internet sites to which the teacher can refer for lesson ideas, instructional strategies, or topical information.

As you observe teachers and talk with them about their mathematics instruction, you will learn about their strengths, weaknesses, and comfort level with certain materials and instructional practices. Teacher observations are scheduled, among other reasons, to help teachers become better at the "craft" of teaching. Staff development offerings, discussed in the following section of this publication, are another way to help teachers develop their knowledge and skills.

Notes, Reminders and Ideas:

Section 7:
Educating and Supporting Teachers

In the past, mathematics instruction was often characterized by a fixed set of routine skills and seemingly arbitrary rules. Under this model, classroom instruction frequently followed a predictable script: (1) review previous material, (2) demonstrate how to solve problems for the day, (3) practice problems, and (4) correct seatwork and assign homework (Stigler and Hiebert 1999). This script reflects the traditional orientation of mathematics teaching where students spent most of their time acquiring isolated skills through repeated practice (Stigler and Hiebert, 1999). Unfortunately, this script for teaching mathematics is still common in some classrooms today.

Today, effective mathematics teaching and learning looks very different from this description. Most noticeably, teachers are expected to teach for meaning and understanding rather than procedural memorization. But for this to happen, schools must establish strong professional development systems and provide teachers with the training they need to improve their mathematics content knowledge and instructional methods.

It may take pushing and prodding on the part of the principal to encourage and persuade some teachers to change their pedagogical style, but the effort will lead to higher levels of student engagement, understanding, and learning. As Neuman and Mohr suggest, the principal's role is to "acknowledge [teachers'] fears, to gently remind them that the old way was not working, and to keep moving forward" (2001, 47).

Developing a Schoolwide Professional Development Plan

Principals who want to raise mathematics achievement must develop support systems for teachers as they work toward continuously improving their own understanding of mathematics content and teaching techniques. Although many school factors combine to influence school achievement, research has consistently shown that a positive

relationship exists between high-quality, targeted staff development and student achievement (Killion undated). As Joyce states, "There is no question that staff development can raise student achievement when it addresses the academic content that teachers teach, their teaching repertoire, and the amount of practice they provide students in particular areas" (in Sparks 2000). To be successful, staff development must occur in a system that supports high levels of learning for both staff and students.

In general, professional development offerings should vary in content, form, and method of presentation. Just like students, teachers have different learning styles and appreciate variety in format and presentation methods. Options include study groups, collaborative teams, individual projects, peer observations, demonstrations, apprenticeships, classroom research projects, observations and feedback from those who are more expert, and pilot programs. Offering a variety of professional development activities will meet individual needs better than a "one-size-fits-all" approach (Learning First Alliance 2000).

A recent study of approximately 1,000 mathematics and science teachers explores the effects of different characteristics of professional development on teachers' learning. Results indicate that teachers benefit most from sessions that feature collective collaboration; emphasize content, active learning, and coherence; and convene at regular intervals over an extended period of time (Garet, et al., 2001).

Participation in professional development offerings should be a motivating experience for teachers, giving them new insight into both the teaching and learning process and their own reasoning and adding to their understanding of how children learn. Teachers should be able to see a clear connection between what they are learning during workshop experiences and how this translates into the daily routines of their classrooms. A principal in Queens, New York, writes,

> The connection between staff learning and student learning is direct and intense, and can't be overestimated. And in order for teachers to be able to offer learning opportunities to kids, they need to experience it themselves first.... A good part of my time and energy in the initial years of the school was spent organizing faculty into small collaborative groups, in a way that allowed them to experience the benefits as well as the difficulties of working together on an area of common need. Having experienced the benefits and the difficulties of small group collaborative learning, the staff can then offer it to students much more effectively, and in more powerful ways. And, in fact, working with kids in the classroom informs their practices with their colleagues on committees and on instructional teams (Neuman and Mohr 2001, 47-48).

As school leaders, principals need to encourage and push teachers to become involved in designing, planning, conducting, and evaluating their own professional

development. Neuman and Mohr characterize the effective leader as "a catalyst for action—a person who appreciates the talents of each teacher and who ensures that appropriate meetings take place, that professional development is well designed, and that there are always new challenges and substantial questions to contemplate and discuss" (2001, 49).

Creating Opportunities for Teacher Collaboration

Given the findings of the Garet et al. (2001) study described previously, principals should try to offer as many opportunities as possible for teachers to participate in collaborative groups that feature ongoing learning activities. Although teacher-to-teacher sharing and support to improve student learning happens informally every day in schools across the country, many schools and districts have developed more formal approaches that capitalize on the expertise of their teachers. Following are brief descriptions of several of these approaches.

Teachers Learning from Teachers

Recently, more schools and districts have begun to experiment and experience success with teacher-to-teacher staff development. In this approach, one or more "teacher leaders" receive training from a district curriculum coordinator or some other individual and then use this knowledge to conduct regularly scheduled staff development sessions for colleagues in their school. Research has shown this model to be an efficient means of spreading knowledge and skills throughout a building, especially in the use of manipulatives for mathematics instruction (Daane 1996).

Peer Coaching

Peer coaching typically involves two experienced teachers who observe and learn from each other's teaching (Ackland 1991). Showers and Joyce characterize the "collaborative work of peer coaching teams [as] much broader than observations and conferences....Teachers learn from each other while planning instruction, developing support materials, watching each other work with students, and thinking together about the impact of their behavior on their students' learning" (1996, 15).

Through a careful pairing of personalities and expertise, as well as some initial training in critical observation, analysis, and conferencing, peer coaching can promote a forum for addressing instructional problems, sharing successful practices, transferring training from workshop to workplace, and reflecting on practice (Robbins 1991).

Study Groups

When teachers collaborate with peers in study groups, they work together to improve learning for their students, while simultaneously fostering growth in their own and their peers' understanding of effective teaching strategies (Cramer, Hurst, and Wilson 1996). Study groups often consist of small communities of teachers and administrators who support one another as they study academic content and teaching strategies. Teams

can address either their individual members' professional growth needs or methods for implementing school or district initiatives (Joyce and Showers 1996).

Making Professional Development Part of Teachers' Workday

Time is often the toughest challenge principals and teachers encounter as they attempt to make room for professional development activities in the already-crowded school schedule. Watts and Castle (1993, in Corcoran 1995) outline five approaches that have been used to create more time for professional development:

- Making better use of available time.
- Using substitutes or releasing students. Some schools are effectively using one morning or afternoon a week for teacher development and other improvement activities.
- Purchasing teacher time by compensating for weekend and summer work.
- Providing common planning time for teachers working with the same children or teaching the same grade on a regular basis.
- Restructuring time by permanently altering teacher responsibilities, the teaching schedule, the school day, or the school calendar.

By building time for professional development into the regular school day, you convey a message about the importance of continuous and ongoing learning. Teachers will make an increased effort to participate in such offerings when they perceive the value you attribute to the activities (DuFour 1999).

Evaluating Your Staff Development Program

In recognition of the need to provide teachers with ongoing learning opportunities, CONNECT, a National Science Foundation-funded, statewide systemic initiative for mathematics and science, has developed the guide *Professional Development Criteria: A Study Guide for Effective Professional Development.* This resource explores the variety of factors that influence quality professional development programs, including the program's context, process, and content goals. Because the document is intended to be used as a study guide, questions are provided throughout to assist school and district leaders in examining their current practices. It is available online at http://bcn.boulder.co.us/connect/publications/prof_dev.pdf

The Context for Professional Development

Teachers are more likely to improve student achievement in mathematics when these conditions prevail:

- *Everyone who affects student learning is involved.* It is largely ineffective to educate classroom teachers about mathematics instruction unless administrators, specialists, teaching assistants, tutors, and parents operate with similar concepts and practices.
- *Student standards, curricular frameworks, textbooks, instructional programs, and assessments are closely aligned with one another.* When academic standards, curricular frameworks, textbooks, instructional programs, and assessments are aligned, teachers can more readily commit effort and resources to implementing them.
- *Professional development is given adequate time and takes place in school as part of the workday.* Teaching children mathematics is a complex activity that is learned with knowledge, coaching, and experience. Because teachers are professionals who do more than manage a room full of students, they need time to reflect on the success of their lessons with others who are working toward similar goals.
- *The expertise of colleagues, mentors, and outside experts is accessible and engaged as often as necessary in professional development programs.* Positive schoolwide change requires collaboration of faculty, administration, and community toward a commonly held vision. Regular (weekly to bimonthly) collaborations with grade level teams, specialists, and facilitators can be seminars in curriculum development, interpretation of student assessments, or acquisition of teaching skill.
- *Strong instructional leadership is present.* Vocal and visible commitment from building administrators is necessary to support improvement of mathematics instruction. These leaders must also cultivate school board and community support for specific goals and practices in mathematics instruction.
- *There is commitment to a long-range plan with adequate funding.* Preparation for change, change itself, and institutionalization of change in teaching practices may take three to five years. Short-term solutions to long-term challenges will not work.

Source: Adapted from Learning First Alliance 2000.

What Kind of Professional Development in Mathematics Instruction Do Teachers Need?

Teaching for understanding is a complex task. Killion (1998) writes,

> [It] relies on teachers' ability to see complex subject matter from the perspectives of diverse students. Teachers' ability to design questions, select instructional and assessment tasks, evaluate student learning, and make instructional, curricular, and assessment decisions depends on how well they understand the content they are teaching (online).

To teach for understanding, elementary and middle school teachers must have:

- An understanding of mathematical concepts and procedures, including concepts of number systems and number sense, geometry, measurement, statistics and probability, and functions and use of variables;
- The ability to present mathematical concepts and procedures in a variety of ways;
- The ability to reason mathematically, solve problems, and communicate mathematics effectively at different levels of formality;
- The ability to make mathematical connections within the discipline and to its uses in the world;
- An awareness of the changes in the nature of mathematics instruction and the ways we teach, learn, and do mathematics;
- The disposition to do mathematics and the confidence to learn mathematics; and
- Fluency in mathematical language and symbolism (CONNECT 1997).

Kober identifies three priorities for staff development in mathematics, which build upon and address many of the abilities described previously. The first concerns the nature of instruction and the changing role of the teacher from a "dispenser of knowledge" to a "facilitator of learning," as called for by the National Council of Teachers of Mathematics. Under this new model, a priority for professional development must be "to help teachers make instruction more active and student-centered, develop students' higher order skills, and encourage students to explore" (1991, 43). This goal for staff development programs is consistent with the current notion that mathematics instruction should challenge students' "power to analyze, reason, and comprehend" (43).

A second priority is to ensure that teachers remain up-to-date on changes in mathematics and education. Professional development experiences that aim to fulfill this goal might address the integration of calculators and computers into mathematics instruction,

use of data for decision making, and the development of hands-on active learning experiences. Other instructional issues that may need to be addressed include the use of manipulatives to engage students in active learning, skills to diagnose students' prior knowledge, and the implications of children's informal mathematics concepts (CONNECT 1997).

The third priority identified by Kober (1991) is to assist teachers in communicating "high expectations for all students—including female, minority, disabled, and disadvantaged students"—and modifying or differentiating instruction to meet the learning needs of each student (43).

An important foundation for each of these priorities is the teachers' knowledge and understanding of mathematics content. Without this necessary element, these instructional priorities will have little impact on children's mathematics achievement.

Teachers are likely to benefit from staff development offerings that address specific instructional topics. The following represent a sample of the most commonly cited instructional areas in which teachers ask for assistance:

- The role of number sense, counting, and manipulative materials;
- Language and its affect on early mathematics learning;
- The implications of children's informal mathematics concepts about size, shape, and space as well as number and chance;
- The role of calculators and computers;
- Probabilistic and proportional reasoning;
- The role of variable and function;
- The inclusion of discrete mathematics; and
- The assessment of mathematics, both informally and formally (NCTM 1991).

Leading the Staff Development Process: The Principal

The effective school principal recognizes the key role staff development must play in developing a learning-focused, high-achieving school. You should take the lead in ensuring that teacher staff development provides opportunities to build a knowledge base, observe models and examples, reflect on one's own practice, change one's own practice, and gain and share expertise. You should also ensure that staff development offerings focus on student learning. This often involves studying the curriculum; reviewing district, state, and national standards; and analyzing the scope and sequence of the curriculum (Killion 1999).

Project TEEM

The Teacher Enhancement in Elementary Mathematics Project (Project TEEM) began as a grassroots movement among community leaders in the Cleveland Municipal School District (CMSD) to address the unacceptably low levels of student performance in mathematics throughout the district. The program aims to improve student achievement in mathematics by providing teachers with the training and resources they need to implement a high-quality program.

TEEM uses a "train-the-trainer" approach to staff development in which teams (two each) of teachers (designated "teacher leaders") in each elementary school in the district were provided with training in mathematics content, instructional methodology, and leadership skills. After four years, these teams of teachers were tasked with providing the other teachers in their buildings with a combination of formal and informal professional development experiences and peer assistance. University professors collaborated with the teacher teams throughout the peer-training process.

Project TEEM has enjoyed much success since its inception in 1994. For example, 80 percent of participating lead teachers have reported that nearly all of the teachers in their buildings have replaced drill and practice with standards-driven, inquiry-based instructional methodology. Additionally, fourth grade students in CMSD have moved the district from the bottom to the top-performing urban district in Ohio on the fourth grade Mathematics Proficiency Test.

A full copy of the final report on Project TEEM is available online at http://cleveland-ed-fund.org/teem/index.htm

Although teachers should take part in identifying professional development objectives and planning activities, the principal must "take responsibility for the big picture and see the magnitude of reciprocal learning throughout the school" (Neuman and Mohr 2001, 49). The larger goal should be attaining a "cyclical model" of professional development in which facilitators learn from participants and participants learn from facilitators (Neuman and Mohr 2001, 49). Such a model includes times for reflection, feedback, and critique so both teachers and school leaders are able to grow professionally.

In the model of staff development illustrated on the following page, both the principal and the teachers play an active role. Together, educators can create a staff development program that is collaborative; authentic; reflective; and, perhaps most importantly, designed to speak to student learning needs.

First Steps in Developing a Mathematics Program for Your Staff

As you design and implement a staff development program in mathematics for your staff, consider the following questions. Your answers to these questions will help you identify the strengths of your current professional development program and the areas in which you may need to focus or redirect attention.

- Do educators see their own growth affecting increased achievement for all students in mathematics?
- Does professional development increase achievement for all students in mathematics?
- Does professional development support mathematics standards at the national, state, and local levels?
- Does professional development support the alignment of curriculum, instruction, and student assessment with mathematics content standards? (CONNECT 1997)

You might begin by determining the staff's needs for mathematics content knowledge improvement at each grade level. Next, outline a plan for providing teachers with time and guidance in developing their own mathematics skills. Too often, mathematics skill development in content is focused on high school mathematics teachers, but elementary and middle school teachers also need opportunities to update their content knowledge skills. Communicate to staff that mathematics learning is ongoing and sustained by collaborating with grade level teams of teachers to identify new mathematics goals. Inform teachers about strategies such as classroom action research projects for implementing and assessing their own development. Make time each year to report incremental mathematics goals that were accomplished and to make plans for new goals. Through these actions, you communicate that teachers' mathematics professional development is an important ongoing and supported process in the school.

Leadership Roles in Professional Development		
Phases	**School Administrator's Role**	**Teacher's Role**
Building a knowledge base	Provides an environment rich in opportunities for personal and professional growth	Takes advantage of the opportunity to participate in those activities provided by the school
Observing models and examples	Creates a nurturing climate whereby teachers are able to observe, model, and share expertise	Willing to share their expertise through modeling, providing examples, and best teaching practices
Reflecting on your own practice	Reflects on his/her own instructional and administrative decision making processes that affect the academic, and personal well-being of all stakeholders	Reflects on his/her own instructional practices as they relate to the academic growth and personal well-being of students
Changing your practice	Assesses the processes that have determined the academic, professional, and personal changes related to systemic change	Assesses his/her instructional practices and makes the necessary changes for a more effective learning environment
Gaining and sharing expertise	Applies the knowledge, maintains the learning environment, and expands the possibilities	Applies the knowledge, maintains the academic growth, and expands the possibilities

Source: Riggs and Gil-Garcia 2001.

Notes, Reminders and Ideas:

Section 8:
Working with Families

Parents are their children's first teachers and are in a prime position to have an important influence on their children's academic development. Through the values they communicate about education, effort, persistence, and responsibility, parents influence their children's mathematics achievement.

Kober asserts that parental attitude toward mathematics is a good predictor of children's mathematics achievement at all grade levels. She cites research that shows that "children's self-concept and confidence in their own mathematics aptitude is more directly related to their parents' perceptions of their competence than to children's own achievement record" (1991, 47).

Parents who are actively involved in their child's education report having a better relationship with their child's school and a higher level of satisfaction with the education their child is receiving. Additionally, their children are absent less often, highly motivated, and experience higher achievement levels (Kober 1991).

Given this, it is important for schools to form strong relationships with families. School leaders, as well as teachers, must work together to encourage parents to create a home environment that encourages learning, express high (but realistic) expectations for their children's achievement and future careers, and become involved in their children's education at school and in the community (School-Home Links 2001).

This chapter explores the benefits of strong home-school partnerships in general, as well as the way in which schools and families can collaborate to promote mathematics achievement. Appendix C contains activities and handouts that can be distributed to families to explain their important role in their child's education and to provide suggestions for activities that can be done at home to reinforce mathematical concepts.

Online Resource to Inform Parents

Recognizing the importance of parental involvement in the development of mathematical thinking, the United States Department of Education has produced the guide *Helping Your Child Learn Math.* The guide discusses what it means to be a problem solver, to communicate mathematically, and to demonstrate reasoning ability. It also includes many suggestions for activities parents can use to help their child develop mathematical skills. The activities are arranged by level of difficulty and grade level and include a "tip box" as well as an explanation of the mathematical concept behind the activity. A reference list of math-related resources, including Web sites, books, computer software, and magazines is included at the end of the guide. *Helping Your Child Learn Math* is available online at www.ed.gov/pubs/parents/Math/

Approaches to Encourage Family Involvement

As you interact with parents, be sure to explain the school's mathematics course offerings and approach to mathematics instruction in a clear and easily comprehensible manner so parents can understand the activities and instructional approaches their children describe. Encourage teachers to also provide or make available to parents copies of course outlines, assignments, and other materials that may assist them in understanding their child's mathematics program.

Briars (1999) makes several suggestions about the type of information that teachers and principals should make readily available to parents. She suggests that parents should be informed of the basic skills their children are learning; be provided with specific information about how they can help their children; and be given information on curriculum, instruction, and assessment. Remember that effective mathematics instruction today looks much different than when your students' parents were in school; this makes frequent and informative communication between school and home even more necessary.

One way that schools can facilitate communication with families is by sponsoring informational or training sessions for parents on mathematics activities they can do with their children at home. During these sessions, parents receive information about how they can support and encourage their child's development of mathematical skills. Topics of training sessions might include helping a child develop number sense, activities to develop reasoning ability, and facilitating the development of problem solving skills.

In addition, teachers should communicate their expectations for students to parents. Parents often appreciate knowing the standards to which students will be held and the

How Will Mathematics Look in Your Child's Classroom?

Parents may express concern when the way their child is learning mathematics is inconsistent with the way they remember learning mathematics when they were in school. Kanter and Darby (1999) suggest explaining the new approaches to mathematics instruction to parents in terms of how mathematics instruction "looks" in today's classrooms. For instance:

- **Children will be expected to know their math facts:** Children will be learning their math facts with an understanding of how facts relate to one another.
- **Children will be doing more than arithmetic:** Children will be seeing that math is much more than arithmetic (knowing the facts and number operations); it involves estimation, geometry, probability, statistics, and more.
- **Children will be striving to achieve high goals:** Children will be achieving high standards of understanding, complexity, and accuracy set for them by their parents, teachers, schools, and states.
- **Children will be actively involved in the study of mathematics:** Children will be doing tasks that involve investigations. They will be talking and writing explanations for their thinking.
- **Children will be working with one another:** Children will be collaborating to make discoveries, draw conclusions, and discuss math. This discourse will facilitate their understanding of mathematics concepts.
- **Children will be evaluated in a variety of ways:** Teachers will use many different ways to determine if children know and understand math concepts. Some of these will include writing samples, projects, or written tests. Not all evaluation will be the same for every classroom or every child.
- **Children will be using calculators to solve problems:** They will be using calculators not as crutches but as tools to solve more complex problems with bigger numbers than they could do otherwise. Children with good knowledge of math facts, number sense, and reasoning about math will be able to use the calculator most effectively.
- **Children will be using computers:** They will be developing databases, spreadsheets, and computer graphics while solving problems. They will be exploring and using virtual manipulatives (Moyer, Bolyard, and Spikell, 2002).

(Source: Kanter and Darby 1999, online)

expectations for their child's development over the course of the school year. Similarly, teachers should be certain to explicitly explain procedures for homework, borrowing manipulative materials, and expectations for special projects to parents to make familial involvement in these activities as easy as possible.

One way to accomplish this and to keep parents informed is to send home a weekly or monthly newsletter. At the beginning of the year, the first-grade newsletter might contain a timeline of the development of mathematical skills and a brief outline of skills students are expected to master by the end of each academic quarter to remain "on grade-level." Procedures for using the school library can also be included, as well as tips for helping children with their homework. Periodic updates can provide information about class activities, special projects, and upcoming events.

Explaining Mathematics Skills to Parents

The Wisconsin Center for Education Research (WCER) has developed *The Mathematics for Parents Newsletter* program to assist teachers in communicating with parents about how children learn mathematics. Each newsletter addresses one mathematical skill and explains how the skill develops, common misconceptions, and good practices that support development of the skill. The newsletters are intended to be sent home with students as new skill areas are introduced in the classroom during the course of the year. Copies of the newsletters are available online at www.wcer.wisc.edu/MIMS/Parent_Newsletters/

Back-to-school nights and parent-teacher conferences can provide another forum for conveying information about the mathematics program to parents. During conferences, teachers can show parents examples of their child's work, results of assessment measures (such as observational notes or tests), and the activities in which their child has participated in class. Back-to-school night serves as an ideal opportunity for teachers to give parents a "tour" of the room and its mathematical materials and to explain the mathematics program to parents. Seeing the materials used first-hand—such as unifix cubes, cuisenaire rods, pattern blocks, base-10 blocks, color tiles, and hundreds boards—will help parents understand the school's orientation toward mathematics instruction and may give parents ideas for mathematics activities they can do with their child.

Similarly, Kate Kline, a kindergarten teacher, suggests holding an "open house" or "family math night" several times a year to allow parents to experience mathematics in the same way their children experience it at school. She writes, "many teachers [find] that

Partners in Learning: How Schools Can Support Family Involvement in Education

Learn to communicate better.

At times, parents feel that educators talk down to them or speak in educational jargon they do not understand. School signs often seem unwelcoming. Schools should make every effort to reach out and communicate with parents in a clear way and listen to what they have to say. To ensure that all parents have access to information, written material should be concise and easily readable. Schools should be parent-friendly. Some school newsletters for parents include a glossary of terms to help parents understand school improvement efforts. Other schools use regularly scheduled telephone calls to stay in contact with families.

Encourage parental participation in school improvement efforts.

When schools develop improvement plans, families ought to be included at every stage of the process to get their input and to give them a sense of shared responsibility. Many schools, supported by the new Goals 2000: Educate America Act, are now developing such plans. They are working to raise academic standards, improve teaching, make schools safer, introduce computers and other learning technologies into the classroom, and to make many other vitally needed changes. The full involvement of parents and other members of the community is instrumental to the success of these efforts.

Involve parents in decision making.

Schools can give parents a more effective voice by opening up the school governance process so that more parents can participate. Many schools hold evening and weekend meetings and conferences to accommodate families' work schedules.

Give teachers the tools to reach out to families.

Staff development can help teachers to understand the benefits of family involvement and show them how to remove barriers to involvement. It can also explain techniques for improving two-way communication between home and schools, and suggest ways to help meet families' overall educational needs.

Make parents feel welcome.

Often the first time a parent comes to school is when a child is in trouble. Schools can help reduce tensions by making initial contacts with parents friendly and respectful. Schools can also reduce distrust by arranging contacts in neutral settings off school grounds. Home visits by family liaison personnel can be particularly helpful. Some programs have used home-school coordinators to run weekly clubs for parents, helping to build parenting skills and trust between families and

Partners in Learning: How Schools Can Support Family Involvement in Education

schools. Schools might also encourage parents, teachers, and students to meet at the beginning of the school year to agree on goals and develop a common understanding.

Overcome language barriers.

Reaching families whose first language is not English requires schools to make special accommodations. Translating materials into a parent's first language helps, but written communication alone is not enough. Ideally, a resource person, perhaps another parent, should be available to communicate with parents in their first language. Interactive telephone voice-mail systems that have bilingual recordings for families are also useful. In addition, English-as-a-second-language classes for parents and grandparents may be helpful.

Use technology to link parents to the classroom.

Educators can creatively use new technology for voice-mail to homework hotlines to educational CD-ROM programs to get parents more involved in the learning process. For example, voice mail systems have been installed in several hundred schools across the country. Parents and students can call for taped messages that describe classroom activities and daily homework assignments. Audiotapes and videotapes can also be used to enhance communication with parents. These are especially helpful in reaching family members who do not read. Even with all the new technology, teachers and other school staff can still use the old telephone to connect with parents. Schools can help by providing teachers with classroom phones.

Encourage communities to join school-family partnerships.

This can be especially effective in reducing schools safety problems that are connected to problems in surrounding neighborhoods. Parents, community residents, and law enforcement officials can help by joining together in voluntary organizations, friendship networks, and neighborhood watches to solve common problems. Schools and community and religious organizations can help by offering after-school cultural and recreational activities. Community-supported student services have also succeeded when families, schools, and community representatives have made the effort to get involved.

Source: Partnership for Family Involvement in Education. 1998. *Partners In Learning: How Schools Can Support Family Involvement In Education.* Online: http://www.ed.gov/pubs/PFIE/schools.html

parents better understand the value of asking their children to think when they themselves are required to investigate a mathematical problem, make conjectures, listen to others' ideas, and respond to thought-provoking questions from a teacher" (2000, 570).

The Leadership Role of the Principal

It would be hard to find a principal who does not speak positively of the relationship between family involvement and school improvement. But talking about the importance of working closely with families does not always translate into implementation, commitment, and resource allocation.

Strong support for making the family-school partnership a priority must come from the school leader. When administrators make it clear that family involvement in students' mathematics learning is important, teachers are more likely to use family involvement strategies.

Moreover, for family involvement efforts to be successful, they must be a school priority. Transforming the school into an institution that treats families as partners in the students' education will take time, and the effort will not always go smoothly. The principal who leads family involvement efforts in the school will need to monitor and nurture the effort continuously.

Above all, schools, under the leadership of principals, possess the primary responsibility for initiating school-family partnerships. Schools need to invest in professional development that supports family involvement, create time for staff to work with parents, supply necessary resources, design innovative strategies to meet the needs of diverse families, and provide useful information to families about how they can contribute to their children's learning (Funkhouser and Gonzales 1997).

Notes, Reminders and Ideas:

Section 9:

Moving Ahead with Your School's Efforts to Improve Mathematics Instruction

The standards and accountability movement has had a dramatic impact on all levels of American education. In schools and classrooms across the country, educators are working to raise the achievement of all students to higher levels. This is true especially in the area of mathematics, because national and international assessments of students' mathematics knowledge have revealed that U.S. students are not the mathematical thinkers or problem solvers they should be.

Goldsmith explains that "as instructional leaders of their schools, principals can contribute to efforts to improve mathematics education in three important ways: by becoming knowledgeable about the goals and strategies of mathematics education reform, by guiding and supporting school improvement efforts, and by involving parents and community members" (2001, 53). In addition, principals need to set a tone of high-expectations for all students and encourage teachers to use instructional strategies that promote the development of "active and independent mathematical thinkers" (Goldsmith 2001, 53).

To improve students' mathematical achievement, educators must change the mathematics students learn and the ways they learn it. A reform-based mathematics curriculum emphasizes conceptual understanding, strategic competence, procedural fluency, and the ability to communicate mathematical ideas, as well as the development of higher-order thinking. By providing teachers with solid information on high-quality, effective mathematics instruction, you can help to develop a mathematics program that better meets the needs of *all* students in your school.

The development of an effective mathematics program requires a coordinated effort to pull together the "essential elements" of mathematics instruction, including horizontal and vertical curriculum alignment, ongoing assessment, additional assistance, and staff development.

A key part of this process is reviewing all the pieces from a district or schoolwide perspective. The initial self-study should focus on these questions:

- Is our approach to mathematics instruction balanced and research-based? Has it been effectively communicated to all staff so what is intended is actually what is taught?
- Are the curriculum and instructional methods well-integrated and cohesive across the grades?
- Are we providing enough instructional time for students in mathematics?
- Do we have a comprehensive and ongoing approach to identifying students struggling in mathematics and their specific problems–one that ensures that no student will be overlooked for so long that he or she has little chance of catching up?
- Do we have a system developed for intervening and providing intensive support for struggling students that uses a variety of approaches (tutoring, after-school instruction, regrouping by mathematics ability, etc.) and that provides more mathematics instruction and time for these students?
- Is staff development and other support available for teachers? For example, do they know how to recognize signs of mathematics difficulty, can they use assessment data to guide instruction, and do they understand how to teach specific skills?
- Have discussions, staff development, and so on been used to help content-area teachers realize how mathematics can be incorporated into their subject areas?
- Is time available for teachers to collaborate to problem solve, share successful approaches, and discuss how to provide assistance for students struggling in mathematics?

As the instructional leader of your school, you must support the efforts of all teachers to promote students' mathematics skills. You can help by providing resources and time for teachers to build their skills, discuss what works, and collaborate in a schoolwide effort to increase the ability of all students to achieve mathematically.

There are many programs and practices you can implement to improve and foster mathematics achievement. Although some are easier and less time-intensive than others, all contribute to the overall goal: establishing and maintaining a high-quality mathematics

program. The following is a brief list of suggestions designed to help you establish a school structure that supports high achievement for all students.

- Support faculty professional development. Many teachers need time, support, and practice to improve their content knowledge and develop new instructional approaches.
- Ensure that decisions about grouping students do not exclude certain students from participating in a challenging, high-quality, comprehensive mathematics curriculum.
- Establish teacher leaders who can mentor other teachers in mathematics.
- Provide time in the school day for teachers to meet, discuss, and collaborate. This will help with the formation of a "community of learners" among your school staff.
- Establish an effective process for the selection and analysis of curricular materials.
- Closely examine the results of high-stakes assessments and use the data to inform curricular decisions.
- Ensure that curricular materials, technology, and assessments are aligned at all grade levels.
- Commit resources to the maintenance and upkeep of such curricular materials as computers and computer software, manipulatives, calculators, overhead projectors, and transparencies (Goldsmith 2001; Midgett and Eddins 2001).

These eight suggestions are examples of ways in which you can use your leadership to guide your school in the process of developing and maintaining high-quality mathematics programs. As the principal, you can set the tone by modeling enthusiasm for mathematics, encouraging your faculty to try new ways of working with all students, communicating with parents, and encouraging a love of mathematics in each student. Your role is also key in providing teachers with time, resources, and training to work together to help every student achieve success in mathematics.

Notes, Reminders and Ideas:

Glossary

Abstract level of instruction – Instruction that makes no reference to particular or specific examples and does not involve the use of manipulatives or pictorial or visual representations.

Active learning – Learning experiences in which students are actively "engaged" in learning through "making connections" with prior knowledge or through physical activities such as touching, acting, walking, drawing, sorting, etc.

Basic facts – Facts that children are expected to know quickly and without the use of pencil and paper, or any other math tools. Included in this category are addition facts through 10, subtraction facts which are the inverses of addition facts, multiplication facts through 10, and division facts that are inverses of the multiplication facts.

Conceptual understanding – Instruction that is designed to promote conceptual understanding focuses on the development of an understanding of mathematical ideas and processes. Instruction is developed in such a way that children are led to understand *what* they are doing and *why*.

Concrete level of instruction – Instruction that uses concrete, tangible materials to teach or solve problems or develop an understanding of basic operations. After teacher modeling, students are often encouraged to manipulate these materials themselves.

Counting back – A computation strategy used to solve subtraction problems. To "count back," students start at a particular number and count backward.

Counting on – A computation strategy used to solve addition problems. To "count on," students start at a particular number and count forward.

Critical thinking – A skill that involves judging the accuracy, validity, and quality of ideas.

Direct instruction – A highly organized method of instruction that includes the teacher explaining new information, demonstrating its use, guiding children while they practice the new skill, and then having them apply it independently.

Estimation – A reasonable approximation that is calculated without actually computing the numbers given. Estimating a solution before solving a problem is a strategy that can help students determine whether their answer is reasonable after they have performed the calculation.

Manipulatives – Refers to a wide variety of physical materials and supplies that students can use to solve or visualize a mathematical operation. Virtual manipulatives, which students manipulate on a computer, are also growing in popularity and availability.

Mental arithmetic – Computations that are performed in one's head without the use of paper and pencil or any other calculating tools.

Metacognition – A term used to describe what you are thinking about your own thinking. It often refers to an individual's awareness of the different processes he or she is using to understand and solve a problem.

Model –A physical or mathematical representation that is constructed to provide a visual depiction of a mathematical problem.

Number – A number is understood to be the concept that answers the question "how many." The concepts of number include counting, place value, pattern, nature of numbers (even, odd, factors, multiples, relations to other numbers), and the understanding of operations and properties of basic operations.

Number sense – Refers to the ability to compute accurately and efficiently, to detect errors, and to recognize results as reasonable.

Patterns – A consistent arrangement of concrete objects, attributes, symbols, or parts; a rule by which the next member of a sequence can be predicted.

Problem-solving situations – Contexts in which problems are presented that apply mathematics to practical situations in the real world or problems that arise from the investigation of mathematical ideas.

Procedural fluency – Refers to the ability to quickly and accurately solve basic computational problems.

Real-world problems – Quantitative problems that are encountered in everyday life; for example, making change or calculating the sale price of an item.

Skip counting – counting by twos, fives, tens, or other values.

Strategy – A plan for achieving some end or solution; steps to follow for a solution that can be generalized to solve other problems.

Symbolic level of instruction – The use of symbols to represent the concepts learned through concrete experiences.

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Appendix A:
Children's Books with Mathematics Themes

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Appendix B: Mathematics Mnemonics

Mnemonics are helpful metacognitive strategies that can assist students in remembering mathematical processes. As with any skill, introduce each mnemonic through teacher modeling and show students how and when to use it. Students need explicit instruction in the use of each strategy to be able to use it effectively. The following mnemonics are reprinted here from the Math VIDS manual (Allsopp and Kyger 1999, online).

Basic Computations

DRAW–This mnemonic provides students with a strategy for solving addition, subtraction, multiplication, and division problems at the representational (pictorial drawing) level or at the abstract level ("in the student's head"). Students who do not need to draw the solution can bypass the draw process and move directly to solving for the answer.

Discover the sign: *student finds, circles, and says the name of the computational process to be used.*

Read the problem: *student reads equation.*

Answer, or draw tallies and/or circles and check your answer: *student refers to drawing for each operation.*

Write the answer: *student writes the answer to the problem.*

Count on Big–This strategy can be used to solve basic addition problems. Students should place their finger on the number of greatest value, or the "biggest" number, and then count on to calculate the answer.

Example:

$$\begin{array}{r} 5 \\ +\ 4 \\ \hline 9 \end{array}$$

"FIVE, six, seven, eight, nine."

Count Back Big– This strategy can be used to solve basic subtraction problems. Students should place their finger on the number of greatest value, or the "biggest" number, and then count down to calculate the answer.

Example:
$$\begin{array}{r} 7 \\ -\ 5 \\ \hline 2 \end{array}$$
"SEVEN, six, five, four, three, two."

Place Value

FIND– This strategy helps students determine place value at an abstract level.

Find the columns (between the numerals).
Insert the t's
Name the columns (by place value)
Determine the place value of individual numbers.

Example:

45 → t	o →
4	5

"4 tens and 5 ones"

Greater Than and Less Than

Gator or Duck Picture Cue–This strategy makes use of the open mouth of a gator or the open beak of a duck and uses this as a visual cue for how to read and interpret the greater than and less than sign. Students are taught that the gator or duck has to open their mouths wider for the "larger" number.

Example:

- Cut out pictures of gator, duck, or another memorable character who can has its mouth open wide.
- Paste the picture on the end of an ice cream stick.
- Initially use the idea of a gator or duck to teach students how to identify greater than/less than; tell students that the gator or duck has to open their mouth wider for the number that has the greatest value.
- Students use pictures to identify the larger number.
- Teach association of "<" and ">" to pictures.
- Fade pictures to use of symbols only.
- Encourage children to think of pictures when using "<" and ">".
- Provide the pictures as cues for students who need them periodically.

Word/Story Problems

FASTDRAW

Find what you are solving for.
Ask yourself: What is the important information?
Set up the equation
Tie down the sign
Discover the sign.
Read the problem.
Answer, or draw tallies/circles and check.
Write the answer.

RIDE

Read the problem correctly.
Identify the relevant information.
Determine the operation and unit for expressing the answer.
Enter the correct numbers and calculate, then check the answer.

Problem Solving Steps

1) Read the problem correctly
2) Paraphrase the problem orally
3) Visualize, graphically display, or draw a picture of the problem
4) State the problem
5) Hypothesize
6) Estimate
7) Calculate
8) Self-check

SOLVE–This is a general problem-solving strategy that can be applied to a variety of work problems including whole numbers, fractions, and decimals.

Study the problem.
Organize the facts.
Line up the plan.
Verify your plan with computation.
Examine your answer.

Order of Operations

ORDER–This strategy is used to assist students who are initially learning to solve multi-operation problems that require understanding of order of operations. The *Rule of Order* helps students remember which operations need to be solved first. The first letter in each word of the phrase, Many Dogs Are Smelly, stands for one of the basic math operations (multiplication, division, addition, subtraction).

Example:

Observe the problem
Read the signs
Decide which operation to do first
Execute the rule of order (Many Dogs Are Smelly!)
Relax, you're done!

Appendix C:
Handouts for Parents

Help Your Child Achieve in Math

- ❑ **Visit your child's school.** Meet with your child's teacher to see if your child is actively involved in math. Find out how you can help your child better understand math problems.

- ❑ **Set high standards for your child in math.** Make sure your child is mathematically challenged and encourage his or her interest and pursuit of math. By the end of the 10th grade, your child should be expected to have studied algebra and geometry.

- ❑ **Help children see that math is very much a part of everyday life.** From statistics in sports to the sale price of clothing, from the calories in food to the amount of gas needed to travel from one city to another, math is important to us every day. Help your child make these connections in math.

- ❑ **Point out that many jobs require math.** From the scientist to the doctor, from the plant manager to the newspaper salesman, from the computer programmer to the hardware store owner, many jobs require a strong foundation in math. Help your child see that math leads to many exciting career opportunities.

- ❑ **Stimulate your child's interest in technology.** Encourage your child to use calculators and computers to further learning.

- ❑ **Play games that help children develop decision-making and mental math skills.** There are many games sold commercially, such a board games, that involve patterns and probability. Play games from your own family traditions such as counting games and games that keep score. Try schoolyard games such as jump rope, hopscotch, and jacks. Games require children to use strategies to make decisions, solve problems, and develop an understanding about numbers and how to use them (number sense) and computational skills.

- ❑ **Positive attitudes about math will reinforce encouragement.** Your feelings will have an impact on how your children think about math and themselves as mathematicians. Positive attitudes about math are important in encouraging your child to think mathematically.

(Source: Kanter and Darby. 1999. "Help Your Child Achieve in Math." In *Helping Your Child Learn Math.* Washington, D.C.: U.S. Department of Education. Online: www.ed.gov/pubs/parents/Math/)

Checklist for Helping Your Child with Homework

1. Show You Think Education and Homework Are Important.

- ❑ Do you set a regular time every day for homework?
- ❑ Does your child have the papers, books, pencils, and other things needed to do assignments?
- ❑ Does your child have a fairly quiet place to study with lots of light?
- ❑ Do you set a good example by reading and writing yourself?
- ❑ Do you stay in touch with your child's teachers?

2. Monitor Assignments.

- ❑ Do you know what your child's homework assignments are? How long they should take? How the teacher wants you to be involved?
- ❑ Do you see that assignments are started and completed?
- ❑ Do you read the teacher's comments on assignments that are returned?
- ❑ Is TV viewing cutting into your child's homework time?

3. Provide Guidance.

- ❑ Do you understand and respect your child's style of learning? Does he work better alone or with someone else? Does he learn best when he can see things, hear them, or handle them?
- ❑ Do you help your child to get organized? Does your child need a calendar or assignment book? A bag for books and a folder for papers?
- ❑ Do you encourage your child to develop good study habits (e.g., scheduling enough time for big assignments; making up practice tests)?
- ❑ Do you talk with your child about homework assignments? Does she understand them?

4. Talk with Someone at School When Problems Come Up.

- ❑ Do you meet the teacher early in the year *before* any problems arise?
- ❑ If a problem comes up, do you meet with the teacher?
- ❑ Do you cooperate with the teacher and your child to work out a plan and a schedule to fix homework problems?
- ❑ Do you follow up with the teachers and with your child to make sure the plan is working?

(Source: U.S. Department of Education. 1995. "Checklist for Helping Your Child With Homework." In *Helping Your Child With Homework*. Washington, D.C.: Author. Online: www.ed.gov/pubs/parents/Homework/pt10.html)

Let's Explore Math! Activities for the Home

Mathematics is everywhere, and every day is filled with opportunities to help children experience it. The following activities are designed for home use so that you and your child can explore math and have fun at the same time. They are reproduced from the U.S. Department of Education publication, *Helping Your Child Learn Math*, and are available online at: www.ed.gov/pubs/parents/Math/

Fill It Up Grades K-2

Materials: a measuring cup, 4 glasses of equal size, and water

What to do:

1) Pour water at different levels (1/3 cup, 1/2 cup, 3/4 cup and 1 cup) in each glass. Put the glasses next to each other. Ask your child: Are all the water levels the same or different?
2) Ask your child questions to encourage comparison, estimation, and thinking about measurement. Which glass has more water? Which has less?
3) Pour more water into one of the glasses to make it equal to the amount of water in another glass. Move the glasses around so that the glasses that have the same amount of water are not next to each other. Ask your child: Which glasses do you think have the same amount of water?
4) As your child begins to understand more, do activities using different-shaped containers that hold the same amount of substance (water, rice, and popcorn kernals). This helps your child see comparisons, as well as the various capacities of different-sized and –shaped containers.

Money's Worth Grades 1-3

Materials: Coins, grocery store coupons, and a pencil

What to do:

1) Coin clues. Ask your child to gather some change in his or her hand without showing what it is. Start with amounts of 25 cents or less (for first-graders, you can start with pennies and nickels). Ask your child to tell you how much money and how many coins there are. Guess which coins are being held. For example, "I have 17 cents and 5 coins. What coins do I have?" (3 nickels and 2 pennies).
2) Clip and Save. Cut out grocery store coupons and tell how much money is saved with coins. For example, if you save 20 cents on soap, say 2 dimes or 4 nickels.
3) Count the ways. How many ways you can make 10 cents, 25 cents, 30 cents, 40 cents, or 50 cents? You can help your child add the coins in various ways to get different answers.

Get Into Shapes Grades 2-4

What to do:

1) Before going to grocery store, show your child pictures of the following shapes: cone, cylinder, square boxes, and rectangular prism. This will help your child identify them when you get to the store.
2) At the store, ask your child questions to generate interest in the shapes. Which items are solid? Which are flat? Which shapes have flat sides? Which have circles for faces? Which have rectangles? Do any have points at the top?
3) Point out shapes and talk about their qualities and their use in daily life. Look to see what shapes stack easily. Why do they? Try to find some cones. How many can you find? Look for stacks that look like a pyramid. Determine which solids take up a lot of space and which ones stack well. Discuss why space is important to the grocer and why the grocer cares about what stacks well. (More space allows for more products to be stored).

Total It Grades 3-5

Materials: License plates, paper, pencil, and calculator

What to do:

1) As you are traveling in your car, or on a bus, each person takes turns calling out a license plate number.
2) All players try to add the numbers in their heads. Talk about what strategies were used in mental math addition. Where the numbers added by 10's like 2+8? Were doubles like 6+6 added?
3) Try different problems using the numbers in a license plate. For example, if you use the plate number 663M218, ask "Using the numbers on the plate, can you make 5?"

Card Smarts Grades 3-5

Materials: Number cards, pencil, and paper

What to do:

1) How many numbers can we make? Give each player a piece of paper and a pencil. Using the cards from 1 to 9, deal 4 cards out with the numbers showing. Using all 4 cards and a choice of any combination of addition, subtraction, multiplication, and division, have each player see how many different numbers a person can get in 5 minutes. Players get one point for each answer. For example, suppose the cards drawn are 4, 8, 9, and 2. What numbers can be made?

Leading Learning Communities NAESP Standards for What Principals Should Know and Be Able To Do

Standard One: Balance Management and Leadership Roles

Standard Two: Set High Expectation and Standards

Standard Three: Demand Content and Instruction That Ensure Student Achievement

Standard Four: Create a Culture of Adult Learning

Standard Five: Use Multiple Sources of Data as Diagnostic Tools

Standard Six: Actively Engage the Community

NAESP is committed to providing elementary and middle level principals with the resources they need to meet the ever-increasing challenges of their position. Toward that end, we are attempting to connect our publications and activities to the standards identified in our landmark publication, Leading Learning Communities: Standards for What Principals Should Know and Be Able to Do.

This book, What Principals Need to Know About Teaching...Reading, relates directly to Standard Three in our Leading Learning Communities publication. We all know the importance of reading as a fundamental skill for students. It is our hope that this book will provide you with information you need about teaching reading and support you in your efforts to provide leadership in this area for your learning community.

Vincent L. Ferrandino
Executive Director
NAESP

National Association of Elementary School Principals. 2001 "Leading Learning Communities, NAESP Standards for What Principals Should Know and Be Able To Do" www.naesp.org

ELEMENTARY SCHOOL
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